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Dynamic Control of a Manipulator with Passive Joints in an Operational Coordinate Space

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Abstract

In this paper, a method to control the tip position of a manipulator with passive joints, which have no actuators, in an operational coordinate space is presented. The equations of motion are described in terms of operational coordinates. The coordinates are separated into *controlled coordinates* and *compensating coordinates*. The accelerations of the controlled coordinates can be arbitrarily adjusted by using dynamic coupling of the manipulator. The effectiveness of the method is verified by experiments using a two-degree-of-freedom manipulator with a passive joint. The experiments show that the controlled coordinates of the position of the manipulator can be controlled precisely by use of the proposed method.

1. Introduction

The number of degrees of freedom of a conventional manipulator is equal to the number of joint actuators. Since the mass of the actuator of a serial type manipulator is a load for the next actuator, the size of the actuator should increase rapidly from the wrist joint to the base joint. As a result, the base joint must be equipped with a huge actuator compared to the load of the manipulator. In order to decrease the weight, cost and energy consumption of a manipulator, various methods have been proposed for controlling a manipulator which has more degrees of freedom than actuators [1]. However, these methods require special mechanisms (e.g. drive chains, drive shafts, transmission mechanisms) in addition to the basic links and joints. In this paper, a method for controlling a manipulator which has more joints than actuators without using additional mechanisms is presented.

The dynamics of a manipulator has non-linear and coupling characteristics. When each joint is controlled by a local linear feedback loop, these factors result in disturbance. The elimination of such dynamic disturbances has been one of the major problems in the control of a manipulator [2-6]. A design theory for a manipulator arm which has neither non-linearity nor dynamic coupling has also been proposed [7]. However, the effects of these disturbances are available to drive a joint which in itself does not have an actuator. It is observed that the dynamic characteristics are actively used in human handling tasks. For example, when a heavy load is handled, the human wrist joint is not forced but kept free and the inertia of the load is utilized effectively. Such a *"dynamic skill"* will also be significant for robot control.

A synergetic control scheme has been proposed for control of a system including unpowered degrees of freedom [8]. In this scheme, the states and generalized forces of the system are partially programmed, and unknown states and unknown generalized forces are determined by the conditions of dynamic equilibrium. This method was applied to synthesis of biped gait and biped postural stabilization. In the biped system, the degrees of freedom between the foot and the ground are unpowered, and they are controlled indirectly using the dynamic coupling with other powered degrees of freedom.

As a means of controlling a manipulator which has more joints than actuators without additional mechanisms, we have proposed a method of controlling passive joints by using dynamic coupling [9]. We also developed an algorithm for point to point control of the manipulator and applied it to a two-degree-of-freedom manipulator [10]. In this method, a manipulator is composed of two types of joints, active joints and passive joints. Each active joint comprises an actuator and a position sensor (e.g. an encoder). Each passive joint comprises a holding brake and a position sensor. When the brakes of the passive joints are engaged, the active joints can be controlled without affecting the state of the passive joints. When the brakes are released, the passive joints can rotate freely. The motion of the active joints generates accelerations of the passive joints through the effect of the coupling characteristics of manipulator dynamics. The passive joints can be controlled indirectly by using this effect. The total position of the manipulator is controlled by combining these two control modes.

When some joint actuators of a manipulator are exchanged for holding brakes with this method, we can build a light-weight, energy-saving and low-cost manipulator. We can take advantage of these merits by applying it to simple assembly robots, control of redundant manipulators, etc. Space applications (e.g. space manipulators, expansion of space structures) may also be effective.

The control with the passive joints released is an essential part of this method. In [9,10], we controlled the manipulator in joint coordinate space. It means that a desired trajectory is assigned to the passive joints and the motion and torque of the active joints is calculated to realize the desired motion of the passive joints. The motion of the active joints is determined by the desired trajectory of the passive joints and the dynamic coupling among the joints. Therefore the motion of the tip of the manipulator cannot be prescribed. However, the position of the tip in an operational coordinate space, e.g. Cartesian coordinate space, is usually important in practical tasks of the manipulator. In this paper, we propose a method to control the position of a manipulator with passive joints not in joint coordinate space but in an operational coordinate space. In this method, the equations of motion are represented in terms of operational coordinates. The desired accelerations can be generated at the controlled coordinates of the same number as the active joints by using dynamic coupling among the coordinates. The feasibility of the proposed method is experimentally verified by a two-degree-of-freedom manipulator.

2. Joint Coordinate Space Control

This section is a review of the scheme for controlling a manipulator with passive joints in joint coordinate space and an algorithm for point to point control, proposed in [9,10].

2.1 Control of Passive Joints

A manipulator with n degrees of freedom is considered. The equations of motion of the manipulator can be written as follows

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{u} \tag{1}$$

where,

$$\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) + \Gamma\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q})$$

$\mathbf{q} \in \mathbf{R}^n$: joint angle vector
$\mathbf{u} \in \mathbf{R}^n$: joint torque vector
$\mathbf{g}(\mathbf{q}) \in \mathbf{R}^n$: gravity torque vector
$\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}) \in \mathbf{R}^n$: Coriolis and centrifugal torque vector
$\mathbf{M}(\mathbf{q}) \in \mathbf{R}^{n\times n}$: inertia matrix
$\Gamma \in \mathbf{R}^{n\times n}$: viscosity friction matrix

We assume that r ($r \geq n/2$) degrees of freedom of the manipulator are active joints. The n-r degrees of freedom are passive joints which have holding brakes instead of actuators.

The angles of r joints, including all the n-r passive joints, are selected from the components of $\mathbf{q}$ and set as a vector $\psi \in \mathbf{R}^r$ (since $r \geq n/2$, $r \geq n-r$). When $r > n-r$, ψ is composed of the angles of the n-r passive joints and those of the $2r$-n active joints. When $r = n-r$, all the joints represented by ψ are passive. In addition, all the remaining n-r joints are active and their angles are represented as $\phi \in \mathbf{R}^{n-r}$. The torque of the r active joints is expressed as $\tau \in \mathbf{R}^r$. When the passive joints are free, the torque of the passive joints is equal to zero. The components of $\mathbf{q}$ and $\mathbf{u}$ are then rearranged as

$$\mathbf{q} = \begin{bmatrix}\phi\\ \psi\end{bmatrix}\begin{matrix}n-r\\ r\end{matrix} \qquad \mathbf{u} = \begin{bmatrix}\tau\\ \mathbf{0}\end{bmatrix}\begin{matrix}r\\ n-r\end{matrix} \tag{2}$$

Accordingly, $\mathbf{M}(\mathbf{q})$ and $\mathbf{b}(\mathbf{q},\dot{\mathbf{q}})$ are also rearranged and partitioned as follows

$$\mathbf{M}(\mathbf{q}) = \overset{\qquad n-r \qquad\quad r}{\begin{bmatrix}\mathbf{M}_{11}(\mathbf{q}) & \mathbf{M}_{12}(\mathbf{q})\\ \mathbf{M}_{21}(\mathbf{q}) & \mathbf{M}_{22}(\mathbf{q})\end{bmatrix}}\begin{matrix}r\\ n-r\end{matrix}$$

$$\mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) = \begin{bmatrix}\mathbf{b}_1(\mathbf{q},\dot{\mathbf{q}})\\ \mathbf{b}_2(\mathbf{q},\dot{\mathbf{q}})\end{bmatrix}\begin{matrix}r\\ n-r\end{matrix} \tag{3}$$

When (2) and (3) are substituted for (1) we obtain

$$\mathbf{M}_{11}\ddot{\phi} + \mathbf{M}_{12}\ddot{\psi} + \mathbf{b}_1 = \tau \tag{4a}$$
$$\mathbf{M}_{21}\ddot{\phi} + \mathbf{M}_{22}\ddot{\psi} + \mathbf{b}_2 = \mathbf{0} \tag{4b}$$

We assume that the dynamic model of the manipulator is exactly known. $\mathbf{M}_{11}$, $\mathbf{M}_{12}$, $\mathbf{M}_{21}$, $\mathbf{M}_{22}$, $\mathbf{b}_1$ and $\mathbf{b}_2$ can be calculated from the dynamic model if the measured value of the joint angle and velocity at each joint is substituted in $\mathbf{q}$ and $\dot{\mathbf{q}}$ of (3). Furthermore, when desired values $\ddot{\psi}_d$ are assigned to the acceleration $\ddot{\psi}$, (4b) is considered as a linear equation with regard to $\ddot{\phi}$. The coefficient matrix $\mathbf{M}_{21}$ corresponds to the dynamic coupling between $\ddot{\phi}$ and the torque of the passive joints. If $\mathbf{M}_{21}$ is non-singular, (4b) can be solved uniquely for $\ddot{\phi}$ as

$$\ddot{\phi} = -\mathbf{M}_{21}^{-1}\mathbf{M}_{22}\ddot{\psi}_d - \mathbf{M}_{21}^{-1}\mathbf{b}_2 \tag{5}$$

When (5) is substituted in (4a) we obtain

$$\tau = (\mathbf{M}_{12} - \mathbf{M}_{11}\mathbf{M}_{21}^{-1}\mathbf{M}_{22})\ddot{\psi}_d + \mathbf{b}_1 - \mathbf{M}_{11}\mathbf{M}_{21}^{-1}\mathbf{b}_2 \tag{6}$$

If we apply this torque τ to the active joints, the resulting acceleration will be $\ddot{\phi}$ and $\ddot{\psi}_d$.

The open-loop control of (6) is sensitive to disturbances and modelling errors. In order to cause the angle and velocity of the passive joint to follow desired values, the following PID feedback control is applied

$$\ddot{\psi}_d' = \ddot{\psi}_d + K_v(\dot{\psi}_d-\dot{\psi}) + K_p(\psi_d-\psi) + K_i\int(\psi_d-\psi)dt$$

$$(K_v, K_p, K_i : \text{diagonal gain matrix}) \quad (7)$$

Here ψ_d, $\dot{\psi}_d$ and $\ddot{\psi}_d$ are the desired values of angles, velocities and accelerations and ψ and $\dot{\psi}$ are the measured values of angles and velocities. Accelerations $\ddot{\psi}_d'$ obtained in (7) is substituted in $\ddot{\psi}_d$ of (6) and the torque τ is determined.

2.2 Point-to-Point Control

When the holding brakes are engaged(ON), the active joints can be controlled without affecting the state of the passive joints. When the holding brakes are released(OFF), the passive joints can move freely and are controlled indirectly by the coupling torque. The position of the manipulator is controlled by combining these two control modes.

Since the passive joints are controlled with the brakes released and the active joints are controlled with the brakes engaged, the control mode should be changed at least once so all the joints of the manipulator can reach the desired position. Here, mode switching is performed twice and the period of positioning is divided into the following three periods:

(i) ψ joints are fixed (brakes ON)
(ii) ψ joints are free (brakes OFF)
(iii) ψ joints are fixed (brakes ON)

In period (i), the passive joints are fixed and initial acceleration of the n-r active joints is performed; In period (ii), the passive joints are released and positioning of r joints, including all the passive joints, is performed along a desired trajectory; In (iii), the passive joint is fixed again and positioning of the n-r active joints is performed.

The number of passive joints is limited to the number of active joints in the above discussions. When the manipulator has r actuators, r joints can be positioned simultaneously. However, if the brakes are released sequentially, the number of passive joints has no limit and the manipulator with more passive joints than actuators can be controlled.

We applied this algorithm to the same manipulator as used in this paper (Fig.1). The stick diagram in Fig.2 represents an experimental result of PTP control. We investigated the absolute accuracy of positioning. The positioning error of the passive joint was less than 1.3×10^{-3}rad. We also investigated the repetitive precision. The standard deviation of the passive joint angle was 1.7×10^{-4}rad when positioning was performed 100 times. It was confirmed that precise positioning of the passive joint is possible by the proposed control method.

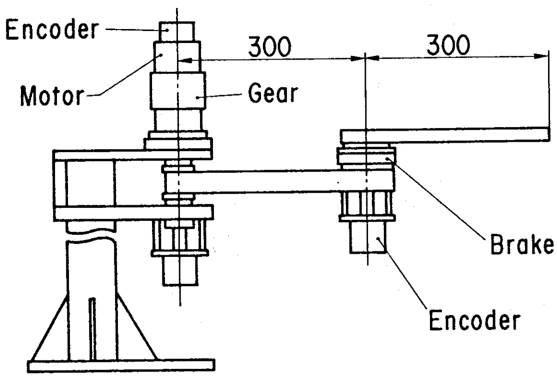


Figure 1 Two-Degree-of-Freedom Manipulator

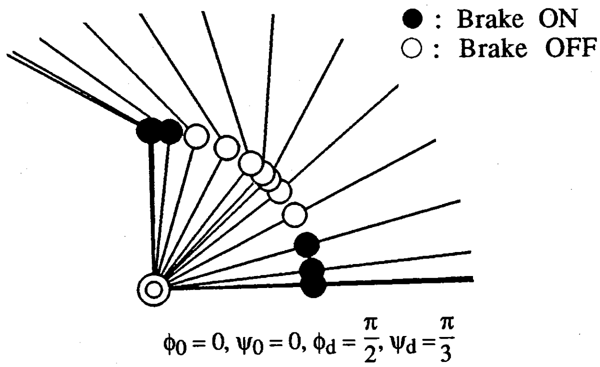


Figure 2 Point-to-Point Control

3. Equations of Motion with Operational Coordinates

In this section, the equations of motion of the manipulator are rewritten in terms of operational coordinates $\mathbf{p} \in \mathbf{R}^n$. We assume that the operational coordinates and the joint coordinates are related as follows

$$\dot{\mathbf{p}} = \mathbf{J}\dot{\mathbf{q}} \quad (8)$$

where, $\mathbf{J} \in \mathbf{R}^{n \times n}$ is a Jacobian matrix. Note that the manipulator has n degrees of freedom and is non-redundant. In the case of a redundant manipulator, some auxiliary coordinates must be added to the operational coordinates so that $\mathbf{J}$ can be inverted. When (8) is differentiated with respect to time, we obtain

$$\ddot{\mathbf{p}} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} \quad (9)$$

If $\mathbf{J}$ is non-singular

$$\ddot{\mathbf{q}} = \mathbf{J}^{-1}(\ddot{\mathbf{p}} - \dot{\mathbf{J}}\dot{\mathbf{q}}) \qquad (10)$$

Here, $\mathbf{p}$, $\mathbf{M}$ and $\mathbf{H} = \mathbf{J}^{-1}$ are partitioned as follows

$$\mathbf{p} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \begin{matrix} n\text{-}r \\ r \end{matrix} \qquad \mathbf{M} = \overset{n}{\begin{bmatrix} \mathbf{M}_1 \\ \mathbf{M}_2 \end{bmatrix}} \begin{matrix} r \\ n\text{-}r \end{matrix}$$

$$\mathbf{H} = \overset{\quad n\text{-}r \quad r}{[\mathbf{H}_1 \ \ \mathbf{H}_2]}\ n \qquad (11)$$

We define components $\mathbf{y}$ of the operational coordinates as *controlled coordinates* and define the remaining components $\mathbf{x}$ as *compensating coordinates*. The desired motion is assigned to the controlled coordinates while the compensating coordinates are controlled so as to realize the desired motion of the controlled coordinates. When eq.(10) and (11) are substituted in eq.(1) we obtain

$$\mathbf{M}_1\mathbf{H}_1\ddot{\mathbf{x}} + \mathbf{M}_1\mathbf{H}_2\ddot{\mathbf{y}} - \mathbf{M}_1\mathbf{H}\dot{\mathbf{J}}\dot{\mathbf{q}} + \mathbf{b}_1 = \boldsymbol{\tau} \qquad (12a)$$
$$\mathbf{M}_2\mathbf{H}_1\ddot{\mathbf{x}} + \mathbf{M}_2\mathbf{H}_2\ddot{\mathbf{y}} - \mathbf{M}_2\mathbf{H}\dot{\mathbf{J}}\dot{\mathbf{q}} + \mathbf{b}_2 = \mathbf{0} \qquad (12b)$$

The equations of motion are represented in terms of the controlled and the compensating operational coordinates. Moreover, they are divided into eq.(12a), which are related to the active joints, and eq.(12b), which are related to the passive joints.

4. Control in an Operational Coordinate Space

In this section, it is shown first that the desired accelerations can be generated arbitrarily at the coordinates of the same number as the active joints. We designed a control system which gives priority to the controlled coordinates. It prescribes a desired trajectory for the controlled coordinates and generates motion of the compensating coordinates in order to realize the desired trajectory of the controlled coordinates.

When measured values of the joint angles and velocities are substituted in $\mathbf{q}$ and $\dot{\mathbf{q}}$ of eq.(12a) and (12b), each component of $\mathbf{M}$, $\mathbf{H}$, $\mathbf{J}$ and $\mathbf{b}$ is determined. If desired values $\ddot{\mathbf{y}}_d$ are assigned to the accelerations $\ddot{\mathbf{y}}$ of the controlled coordinates $\mathbf{y}$, eq.(12b) can be considered as a linear equation with regard to $\ddot{\mathbf{x}}$. If $\mathbf{M}_2\mathbf{H}_1 \in \mathbf{R}^{(n-r)\times(n-r)}$ is non-singular (and hence invertible), (12b) can be solved uniquely as

$$\ddot{\mathbf{x}} = (\mathbf{M}_2\mathbf{H}_1)^{-1}(-\mathbf{M}_2\mathbf{H}_2\ddot{\mathbf{y}}_d + \mathbf{M}_2\mathbf{H}\dot{\mathbf{J}}\dot{\mathbf{q}} - \mathbf{b}_2) \qquad (13)$$

When eq.(13) is substituted in eq.(12a), the torque $\boldsymbol{\tau}$ to realize the desired accelerations $\ddot{\mathbf{y}}_d$ can be determined.

$$\boldsymbol{\tau} = \{\mathbf{M}_1 - \mathbf{M}_1\mathbf{H}_1(\mathbf{M}_2\mathbf{H}_1)^{-1}\mathbf{M}_2\}(\mathbf{H}_2\ddot{\mathbf{y}}_d - \mathbf{H}\dot{\mathbf{J}}\dot{\mathbf{q}}) + \mathbf{b}_1 - \mathbf{M}_1\mathbf{H}_1(\mathbf{M}_2\mathbf{H}_1)^{-1}\mathbf{b}_2 \qquad (14)$$

When we apply this torque $\boldsymbol{\tau}$ to the active joints, we will obtain the desired accelerations $\ddot{\mathbf{y}}_d$. In other words, the accelerations of r controlled coordinates of the operational coordinates of the manipulator can be arbitrarily determined by the torque of the r active joints.

In case of an open loop control, in which the torque is calculated according to eq.(14) with the accelerations of the desired trajectory, it is possible that the manipulator deviates from the desired trajectory due to the disturbances or the modeling error. We designed a closed loop control system to suppress the tracking error. The following PID control is applied

$$\ddot{\mathbf{y}}_d' = \ddot{\mathbf{y}}_d + \mathbf{K}_v(\dot{\mathbf{y}}_d - \dot{\mathbf{y}}) + \mathbf{K}_p(\mathbf{y}_d - \mathbf{y}) + \mathbf{K}_i\int(\mathbf{y}_d - \mathbf{y})dt \qquad (15)$$

where $\mathbf{y}_d$, $\dot{\mathbf{y}}_d$ and $\ddot{\mathbf{y}}_d$ are the desired values of positions, velocities and accelerations of the controlled coordinates respectively. $\mathbf{K}_v$, $\mathbf{K}_p$ and $\mathbf{K}_i$ are the diagonal gain matrix. Accelerations $\ddot{\mathbf{y}}_d'$ of eq.(15) is substituted in $\ddot{\mathbf{y}}_d$ of eq.(14) to determine the torque $\boldsymbol{\tau}$. From eq.(15)

$$(\ddot{\mathbf{y}}_d - \ddot{\mathbf{y}}) + \mathbf{K}_v(\dot{\mathbf{y}}_d - \dot{\mathbf{y}}) + \mathbf{K}_p(\mathbf{y}_d - \mathbf{y}) + \mathbf{K}_i\int(\mathbf{y}_d - \mathbf{y})dt = 0 \qquad (16)$$

The controlled coordinates are guaranteed to converge to the desired values if $\mathbf{K}_v$, $\mathbf{K}_p$ and $\mathbf{K}_i$ are chosen such that all the poles of the system (16) are located in the left-half plane. Fig.3 represents a block diagram of the control system.

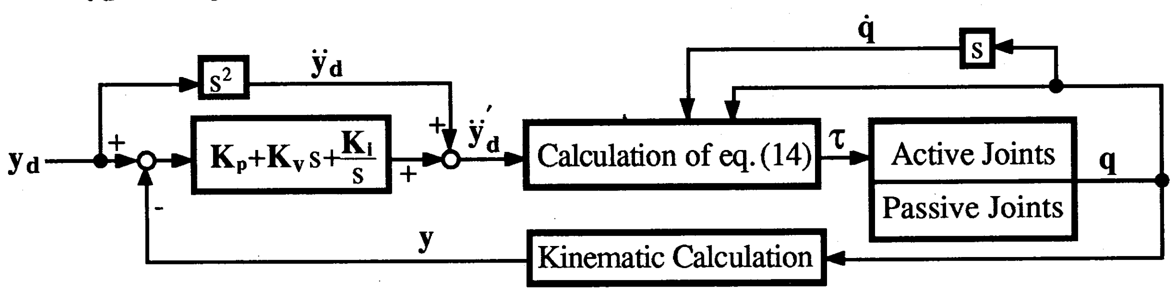


Figure 3 Control System in an Operational Coordinate System

5. Experiments

5.1 Two-Degree-of-Freedom Manipulator

We made experiments of this control method with a two-degree-of-freedom horizontally-articulated manipulator. Fig.1 shows the manipulator. The first axis (ϕ) is an active joint and the second axis (ψ) is a passive joint. The active joint is driven by a DC servo motor with a harmonic drive gear. The brake of the passive joint is an electromagnetic type. Fig.4 shows the model of the manipulator. Table 1 shows the parameters of the model. In the eq.(1)

$$\mathbf{M} = \begin{bmatrix} \frac{1}{3}m_1L^2+\frac{4}{3}m_2L^2+m_2L^2\cos\psi+J_M & \frac{1}{3}m_2L^2+\frac{1}{2}m_2L^2\cos\psi \\ \frac{1}{3}m_2L^2+\frac{1}{2}m_2L^2\cos\psi & \frac{1}{3}m_2L^2 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} -m_2L^2\sin\psi\dot{\phi}\dot{\psi}-\frac{1}{2}m_2L^2\sin\psi\dot{\psi}^2+D_1\dot{\phi} \\ \frac{1}{2}m_2L^2\sin\psi\dot{\phi}^2 \end{bmatrix} \quad (17)$$

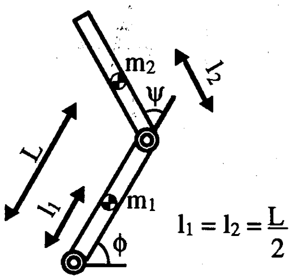


Figure 4 Model of the Manipulator

Table 1 Parameters of the Manipulator

m_1	Mass of link 1	2.0kg
m_2	Mass of link 2	1.0kg
L	Length of link 1 and 2	0.3m
D_1	Viscous friction of the actuator	2.2Nms/rad
J_M	Moment of inertia of the actuator	0.24kgm^2

5.2 Control in Cartesian Coordinate Space

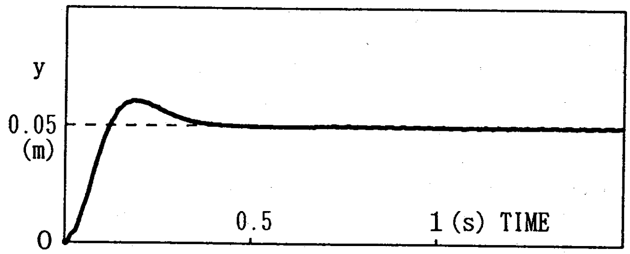
A Cartesian coordinate space is taken as an operational coordinate space. The origin is at the first joint. The coordinate transformation from the joint coordinate space to the operational coordinate space is

$$x = L\cos\phi + L\cos(\phi+\psi)$$
$$y = L\sin\phi + L\sin(\phi+\psi) \quad (18)$$

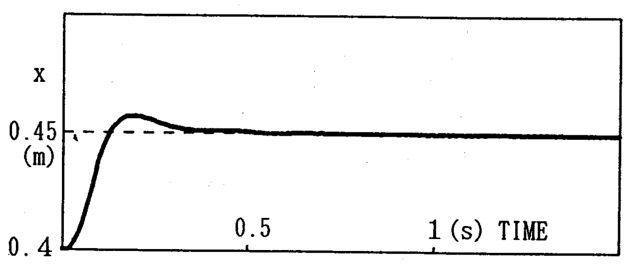
The Jacobian matrix is

$$\mathbf{J} = L\begin{bmatrix} -\sin\phi-\sin(\phi+\psi) & -\sin(\phi+\psi) \\ \cos\phi+\cos(\phi+\psi) & \cos(\phi+\psi) \end{bmatrix} \quad (19)$$

There are two cases; the case in which y is the controlled coordinate and x is the compensating coordinate and the case in which x is the controlled coordinate and y is the compensating coordinate. From the state where the manipulator is at rest, a step change of the reference is given for the controlled coordinate in each case. Fig.5 shows the response. The initial position is x=0.4(m), y=0(m). In Fig.5(a) y is the controlled coordinate and the reference is y=0.05(m). In Fig.5(b) x is the controlled coordinate and the reference is x=0.45(m). The feedback gains are set so that the pole of the system is a triple root. The sampling interval is 2ms with i80386+80387. (In the case of multi-degree-of-freedom, eq.(14) includes $[n^3-2n^2r+nr^2+7n^2-3nr+2r^2]$ multiplications and inverse of $(n-r)\times(n-r)$ matrix. It would be desirable to develop an efficient computation algorithm.) In the result, the measured value of the controlled coordinate (solid line) converges to the reference (dotted line). The error from the reference after the convergence is (a) 0.14mm, (b) 0.08mm. Next, a desired trajectory is assigned for the controlled coordinate. Fig.6 shows the result of the trajectory tracking. The controlled coordinate increases with constant velocity from the stationary state and decreases with constant velocity again in the desired trajectory. Abrupt change in velocity occurs at the beginning of the trajectory and at the moment the direction changes. The measured value (solid line) follows the desired trajectory (dotted line) except just after those moments. The stick diagram (Fig.7) represents the motion of the manipulator, when y coordinate tracks a desired trajectory with constant velocity. The initial acceleration is done with the passive joint fixed. In Fig.7 y coordinate increases constantly.

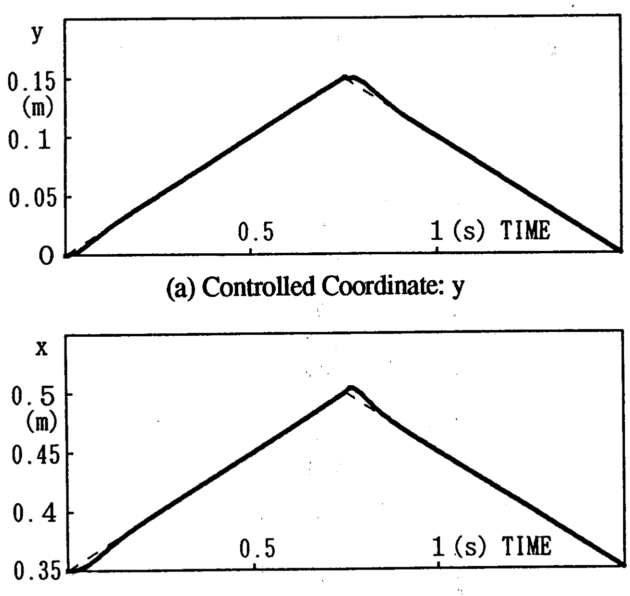


(a) Controlled Coordinate: y



(b) Controlled Coordinate: x

Figure 5 Step Response of the Controlled Coordinate



(a) Controlled Coordinate: y

(b) Controlled Coordinate: x

Figure 6 Tracking of a Desired Trajectory

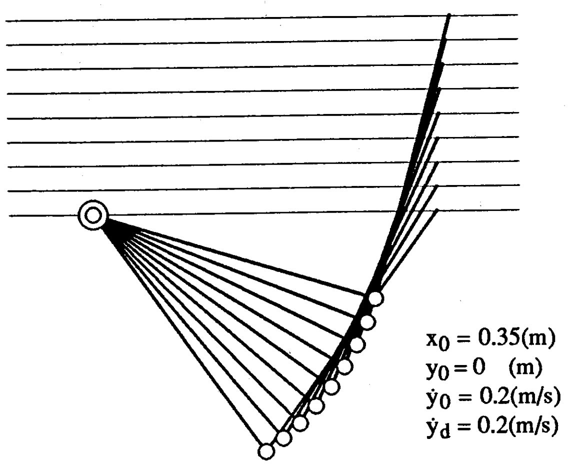
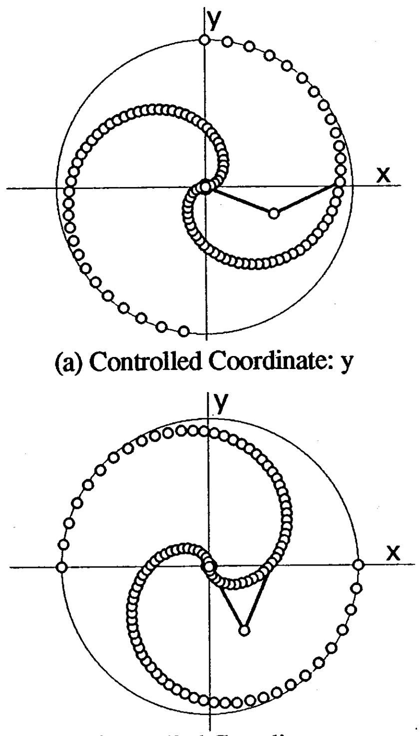


Figure 7 Motion of the Manipulator

5.3 Dynamic Singularity

The condition of realization of this control method is that eq.(12b) has a unique solution. The non-singularity of matrix $\mathbf{M}_2\mathbf{H}_1$ ($\det[\mathbf{M}_2\mathbf{H}_1] \neq 0$) is equivalent to this condition. In the case of a two-degree-of-freedom manipulator, the condition is $\mathbf{M}_2\mathbf{H}_1 \neq 0$. Fig.8 shows the dynamic singular points of the manipulator which is used in the experiments. $\mathbf{M}_2\mathbf{H}_1 = 0$ at those points. In Fig.8(a) y is the controlled coordinate and in Fig.8(b) x is the controlled coordinate.

The acceleration of the compensating coordinate cannot have influence on the acceleration of the controlled coordinate in those positions. The acceleration of the controlled coordinate is determined by the position and velocity of the manipulator irrespective of the acceleration of the compensating coordinate. Therefore this method is difficult to use near these dynamic singular points. Some algorithm to avoid these difficulties is necessary. When Fig.8(a) and (b) are compared, it is observed that these dynamic singular points are distributed alternately. In other words, where the one coordinate is not controllable, the other coordinate is controllable. It is expected that a trajectory can be composed in which controlled coordinates are switched alternately in order to avoid dynamic singularity.



(a) Controlled Coordinate: y

(b) Controlled Coordinate: x

Figure 8 Location of Dynamic Singular Points

5.4 Modelling Error

This method depends essentially on a dynamic model of the manipulator. In the experiments of this paper, each parameter of the manipulator was calculated or determined experimentally in advance. However, a load at the tip of the manipulator causes the change of the dynamic parameters. Here, the effect of the modelling error is roughly investigated. From eq.(12a), (12b), (14) and (15) the accelerations of the controlled coordinates are represented as

$$\ddot{y} = \alpha(q)^{-1}\hat{\alpha}(q)\,\{\ddot{y}_d+\mathbf{K}_v(\dot{y}_d-\dot{y})+\mathbf{K}_p(y_d-y)+\mathbf{K}_i\textstyle\int(y_d-y)dt\}+\alpha(q)^{-1}\{\hat{\beta}(q,\dot{q})-\beta(q,\dot{q})\} \qquad (20)$$

where,

$$\alpha(\mathbf{q}) = \mathbf{M}_1\mathbf{H}_2 - \mathbf{M}_1\mathbf{H}_1(\mathbf{M}_2\mathbf{H}_1)^{-1}\mathbf{M}_2\mathbf{H}_2$$
$$\beta(\mathbf{q},\dot{\mathbf{q}}) = -\{\mathbf{M}_1 - \mathbf{M}_1\mathbf{H}_1(\mathbf{M}_2\mathbf{H}_1)^{-1}\mathbf{M}_2\}\mathbf{H}\dot{\mathbf{J}}\dot{\mathbf{q}} + \mathbf{b}_1 - \mathbf{M}_1\mathbf{H}_1(\mathbf{M}_2\mathbf{H}_1)^{-1}\mathbf{b}_2$$

$\hat{\alpha}(\mathbf{q})$, $\hat{\beta}(\mathbf{q},\dot{\mathbf{q}})$ are the estimates of $\alpha(\mathbf{q})$, $\beta(\mathbf{q},\dot{\mathbf{q}})$ respectively. When the model parameters are complete ($\hat{\alpha}(\mathbf{q}) = \alpha(\mathbf{q})$, $\hat{\beta}(\mathbf{q},\dot{\mathbf{q}}) = \beta(\mathbf{q},\dot{\mathbf{q}})$), this relationship leads to eq.(16). Eq.(20) has basically the same form as the computed torque method of a conventional manipulator which has an actuator for each joint. That is, the proposed control of the controlled coordinates is no less robust than the computed torque method.

On the other hand, the compensating coordinates absorb the modelling error. If the motion of the manipulator is simulated in advance, the trajectory of compensating coordinates deviates from the simulation. In this sense, the proposed method is sensitive to the modelling error. The authors expect that this method may become more effective if it is used together with a real-time parameter identification or adaptive control method [11].

We also investigated the robustness of the control method experimentally. A weight (0.5kg) is attached to the tip of the manipulator. The same experiments of step response as in Fig.5 are done using the parameter without considering the weight. The error of the controlled coordinates after the step response is x : 0.18mm, y : 0.24mm. The increase in the error of the controlled coordinates caused by the modelling error is small.

6. Conclusions

A method to control a manipulator with passive joints in an operational coordinate space has been proposed. The equations of the motion are represented in terms of operational coordinates. The desired accelerations can be generated at the controlled coordinates of the same number as active joints by using dynamic coupling among the coordinates. In the example, position of a two-degree-of-freedom manipulator with a passive joint is represented in a Cartesian coordinate space. One of the two coordinates is controlled to follow desired value. Since closed loop control is included in this method, the controlled coordinates can be controlled precisely even under the presence of disturbances or small modeling errors.

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