Proceedings of the 1994 IEEE International Conference on Multisensor Fusion and Integration for Intelligent Systems (MFI'94) • Las Vegas, NV • Oct. 2-5, 1994

Compensation of Time Lag between Actual and Virtual Spaces by Multi-Sensor Integration

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Abstract—
Unconstrained measurement of human head motion is essential for HMD (Head Mounted Display) to be really interactive. Polhemus sensors developed for that purpose have deficiencies of critical latency and low sampling rate. This paper proposes a method which compensates the latency and raises the effective sampling rate through the integration of Polhemus and gyro sensors. For this purpose, we derived a physically consistent system model of human rotational motion described by quaternion and angular velocity. This integration consists of the optimal prediction based on simultaneous measurement of both sensors, and the extrapolation of the estimate based on measurement of gyro sensor. This paper also reports the implementation of the proposed method.

I. Introduction

Synchronization of user's motion in actual space and virtual space is significant in virtual reality (VR), because under the situation, where time lag between these spaces is greater than 100ms, users tend to feel motion sickness and to feel such virtual environment less interactive.

Motion sensors used in VR systems are classified into two groups. One is the mechanical link-type sensors like BOOM, and the other is the unconstrained sensors like Polhemus Tracker. Latter has advantage of unconstrainedness and wider motion range, but has deficiencies of low sampling rate($\sim$ 60Hz) and critical latency ($\sim$ 100ms). Although Polhemus "Fastrak" is improved in the respect of sampling rate($\sim$120Hz), communication delay ($\sim$40ms with RS232C), and an ad-hoc linear prediction filter, it is still insufficient for interactiveness in the virtual environment.

Most VR systems use raw output of their motion sensor in spite of the decrease in interactiveness by this latency, or try ad-hoc filtering in order to compensate the latency such as linear extrapolation from both smoothed measurements and crude estimates of its instantaneous differential of user's motion.

Friedmann et al. [0] pointed out that by such ad-hoc approach user's quick motion causes poor prediction which overshoots the actual motion and that users are forced to make slow deliberate motions. They solved this problem on translational motion by means of the prediction based on the optimal linear estimation theory. However they ignored rotational motion, which is far more critical for HMD. Liang et al.[0] proposed to adopt the equivalent angle-axis representation for rotational motion and to apply four independent Kalman filters. These four system models did not include angular velocity explicitly in state vector.

This paper proposes a method which compensates the latency and raises sampling rate through integration of multi-sensor information. Our approach is to compensate the latency of measurement of absolute rotation (quaternion) by optimal prediction filter and to extrapolate the interval based on the measurement of relative rotation (angular velocity in body coordinate). We derived the physically consistent system model to integrate information of both absolute and relative rotation.

II. Multi-sensor integration

The goal of our research is to acquire the sensing system, whose measurement delay is negligible, and whose accuracy and estimate rate is higher than ordinary unconstrained motion sensors for VR.

Prediction is used for compensation of measurement delay. Kalman filter is known well as the method of designing optimal prediction filter with taking account of complex motion of target system and measurement error. This has been applied to the measurement delay of the magnetic tracker [0][0]. Prediction of physical quantity needs some kind of its derivative in state vector. But only position or quaternion was measured and its derivative was not measured in these researches. We expected the improvement of estimates by adding the sensor of the derivative.

This demands the physically consistent system model which describes the relation among physical quantities and its derivatives. Liang et al. [0] adopted equivalent angle-axis representation, and constructed four independent Kalman filters. They used four system model to describe rotational motion of a head. However this should be dealt by one system model. We derived the approximated linear system model, which describes the relation between quaternion and angular velocity explicitly. This system model enables the integration of magnetic sensor and gyro sensors by Kalman filter.

Kalman filter has only one measurement model. If we integrate sensors of various sampling rates by Kalman filter, the estimate rate of the filter is determined by the slowest sensor at most. If we want to improve the estimate rate, the information of quick sensor should be reflected to the estimates rationally in some way.

In our problem, the slowest sensor is the magnetic tracker and the quick sensor is gyro sensor. We propose: 1) extrapolation by integrating the displacement of quaternion calculated from the measured angular velocity when only the measurement of angular velocity is available, and 2) the optimal prediction by Kalman filter when the measurement of both quaternion and angular velocity is available.

A. *Modelling*

We adopt quaternion [0] instead of Euler angle as the representation of orientation. The reasons are : 1) when a human body rotates over 2π radian, Euler angle exceeds the measure range of the magnetic tracker, $-\pi \sim +\pi$ and the management around the boundary is complex, and 2) the system model of human rotational motion described by Euler angle has singular points.

It is known that rotation is also represented by complex 2×2 matrix in general as 3×3 rotational matrix [0]. This matrix consists of 4 real numbers $q_0 \sim q_3$.

$$Q = \begin{pmatrix} q_0 + iq_3 & q_2 + iq_1 \\ -q_2 + iq_1 & q_0 - iq_3 \end{pmatrix} \tag{1}$$

Where

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1 \tag{2}$$

We call $q_0 \sim q_3$ quaternion. Rotations around Z, Y, X axis each are represented by these 2×2 matrices.

$$Q_z(\gamma) \equiv \begin{pmatrix} e^{i\gamma/2} & 0 \\ 0 & e^{-i\gamma/2} \end{pmatrix} \tag{3}$$

$$Q_y(\beta) \equiv \begin{pmatrix} \cos\beta/2 & -\sin\beta/2 \\ \sin\beta/2 & cos\beta/2 \end{pmatrix} \tag{4}$$

$$Q_x(\alpha) \equiv \begin{pmatrix} \cos\alpha/2 & i\sin\alpha/2 \\ i\sin\alpha/2 & \cos\alpha/2 \end{pmatrix} \tag{5}$$

If we rotate the coordinate by ZYX Euler angle γ, β, α, corresponding 2×2 matrix is given by

$$Q = Q_x(\alpha)Q_y(\beta)Q_z(\gamma) \tag{6}$$

Therefore in terms of ZYX Euler angle γ, β, α, the quaternion is given by

$$q_0 = \cos\frac{\gamma}{2}\cos\frac{\beta}{2}\cos\frac{\alpha}{2} + \sin\frac{\gamma}{2}\sin\frac{\beta}{2}\sin\frac{\alpha}{2} \tag{7}$$

$$q_1 = \cos\frac{\gamma}{2}\cos\frac{\beta}{2}\sin\frac{\alpha}{2} - \sin\frac{\gamma}{2}\sin\frac{\beta}{2}\cos\frac{\alpha}{2} \tag{8}$$

$$q_2 = \cos\frac{\gamma}{2}\sin\frac{\beta}{2}\cos\frac{\alpha}{2} + \sin\frac{\gamma}{2}\cos\frac{\beta}{2}\sin\frac{\alpha}{2} \tag{9}$$

$$q_3 = \sin\frac{\gamma}{2}\cos\frac{\beta}{2}\cos\frac{\alpha}{2} - \cos\frac{\gamma}{2}\sin\frac{\beta}{2}\sin\frac{\alpha}{2} \tag{10}$$

Let $\omega = (\omega_z\, \omega_y\, \omega_x)$(rad/sec) be angular velocity in body coordinate. Let $q_0 \sim q_3$ be the quaternion at time t, $p_0 \sim p_3$ be the quaternion at time $t + dt$, and dQ be the 2×2 matrix corresponding to the rotation between t and $t + dt$. dQ is given in terms of ω by

$$dQ \simeq \begin{pmatrix} 1 + i\frac{\omega_z}{2}dt & \frac{\omega_y}{2}dt + i\frac{\omega_x}{2}dt \\ -\frac{\omega_y}{2}dt + i\frac{\omega_x}{2}dt & 1 - i\frac{\omega_z}{2}dt \end{pmatrix} \tag{11}$$

$dQ\,Q$ is given in terms of $p_0 \sim p_3$ by

$$dQ\,Q = \begin{pmatrix} p_0 + ip_3 & p_2 + ip_1 \\ -p_2 + ip_1 & p_0 - ip_3 \end{pmatrix} \tag{12}$$

Define F' as below.

$$F' \equiv \begin{pmatrix} -q_3 & -q_2 & -q_1 \\ q_2 & -q_3 & q_0 \\ -q_1 & q_0 & q_3 \\ q_0 & q_1 & -q_2 \end{pmatrix} \tag{13}$$

Combining (11) and (12), and using (13), we obtain the process dynamics expressed by quaternion and angular velocity in body coordinate.

$$\begin{aligned} &(\, p_0 \quad p_1 \quad p_2 \quad p_3 \,)^T \\ &= (\, q_0 \quad q_1 \quad q_2 \quad q_3 \,)^T + F'\omega\frac{dt}{2} \end{aligned} \tag{14}$$

Apparently there is no singularity.

Let dt be unit sampling interval and T be latency of measurement of quaternion. State vector consists of quaternion and angular velocity. We use the notation of

$$q \equiv (\, q_0 \quad q_1 \quad q_2 \quad q_3 \,)^T \tag{15}$$

$$\omega = (\, \omega_z \quad \omega_y \quad \omega_x \,)^T \tag{16}$$

By this notation state vector x is given by

$$x = (\, r^T \quad \omega^T \,)^T \tag{17}$$

System model is given by (19), where $\boldsymbol{w}_n$ is a zero mean white noise of covariance Q. Let H be measurement matrix corresponding to simultaneous measurement of both quaternion and angular velocity. The measurement model corresponding to the simultaneous measurement is given by (22), where $\boldsymbol{v}_n$ is an additive measurement error and R is its covariance.

$$F \equiv \left(\begin{array}{c|c} I & \frac{dt}{2}F' \\ \hline 0 & I \end{array}\right) \tag{18}$$

$$x_{n+1} = F\,x_n + \boldsymbol{w}_n \tag{19}$$

$$\overline{\boldsymbol{w}_m\,\boldsymbol{w}_n^T} = Q\,\delta_{mn} \tag{20}$$

$$H \equiv \left(\begin{array}{c|c} I & \frac{-t}{2}F' \\ \hline 0 & I \end{array}\right) \tag{21}$$

$$y_n = H\,x_n + \boldsymbol{v}_n \tag{22}$$

$$\overline{\boldsymbol{v}_m\,\boldsymbol{v}_n^T} = R\,\delta_{mn} \tag{23}$$

B. *Algorithm*

The algorithm [1] is divided into 2 procedures. Procedure 1 is for the extrapolation of the state vector based on the measurement of angular velocity only. Procedure 2 is for the optimal prediction based on the simultaneous measurement of both quaternion and angular velocity.

Procedure 1

Substitute angular velocity in state vector x_n of mean angular velocity within previous 30ms

$$\overline{\omega} = \left(\begin{array}{ccc} \overline{\omega_z} & \overline{\omega_y} & \overline{\omega_x} \end{array}\right) \tag{24}$$

and update as below.

$$x_{n+1/n+1} = \left(\begin{array}{c} r_{n/n} + \delta t \cdot F'\,\overline{\omega} \\ \hline \overline{\omega} \end{array}\right) \tag{25}$$

$$P_{n+1/n+1} = FP_{n/n}F^T + GQG^T \tag{26}$$

Procedure 2

$$x_{n+1/n} = F\,x_{n/n} \tag{27}$$

$$P_{n+1/n} = FP_{n/n}F^T + GQG^T \tag{28}$$

$$x_{n/n} = x_{n/n-1} + K_n[y_n - H\,x_{n/n-1}] \tag{29}$$

$$P_{n/n} = p_{n/n-1} - K_nHP_{n/n-1} \tag{30}$$

$$K_n = P_{n/n-1}H^T[HP_{n/n-1}H^T + R]^{-1} \tag{31}$$

[1] Our method can also be easily applied to rotational motion. As for translational motion, we can deal with each axis X,Y,Z respectively. In case we adopt state vector $(x\ v_x)$, we use

$$F = \left(\begin{array}{c|c} 1 & dt \\ \hline 0 & 1 \end{array}\right) \qquad H = \left(\begin{array}{c|c} 1 & -T \\ \hline 0 & 1 \end{array}\right)$$

System and measurement dynamics are linear equations with constant coefficients. If we assume the driving noise statistically stationary, the filtering process reach a "steady state", where $P_{n/n}$ and K_n converge at constant matrices. This matrices can be calculated beforehand, and the filtering calculation is much simpler than Procedure 2 above.

Procedure 2 is the same as the algorithm of discrete Kalman filter [0].

Fig.1 is the timing chart of available sensors and procedures. We measured the orientation at 25Hz and the angular velocity at 100Hz. The output was obtained at 100Hz.

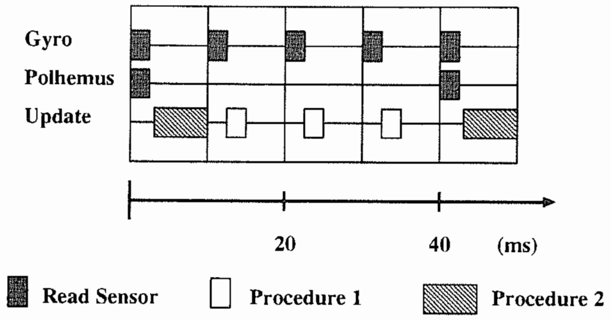

Fig. 1. Timing chart of available sensors and procedures in the proposed method

III. Off-line experiment

A. *Apparatus*

We used Polhemus Tracker as the sensor of orientation, and 3 compact gyro sensors of Murata Co. (GYROSTAR ENC-05S) as the sensor of angular velocity in the body coordinate fixed to the human head.

In order to evaluate performance, we compared the output of the proposed method with measurement of the link-type motion sensor [0]. The precision of the link-type sensor (accuracy 0.0125 deg. resolution 0.025 deg.) was much better than that of Tracker (accuracy 0.1 deg. resolution 0.5 deg.) because rotational angle of each joint was measured by high-precision optical encoder and up-down counter.

The output of the up-down counter was directly connected to DATA Bus of the computer, and lead-time of this up-down counter was only 2 μs. CPU performance determined the delay of this system. This system with 80286 (10MHz) measured operator's motion and controlled the robot at least 300Hz. Therefore the response delay and communication delay of this link-type sensor was negligible compared with Polhemus Tracker ($\sim$ 80 ms). We used measurement of this link-type sensor as the standard signal.

B. *Noise parameter*

We assume process noise and measurement noise to be independent. They are determined by Q and R. Let

I_3, I_4 be 3×3 4×4 unit matrices, and we suppose

$$Q = \left(\begin{array}{c|c} w_1^2 I_4 & 0 \\ \hline 0 & w_2^2 I_3 \end{array}\right) \quad (32)$$

$$R = \left(\begin{array}{c|c} v_1^2 I_4 & 0 \\ \hline 0 & v_2^2 I_3 \end{array}\right) \quad (33)$$

w_1, w_2 are determined as the standard deviation of remnant error when data of the high-precision link-type sensor [0] was applied to system model. We used $w_1 = 0.0212$, $w_2 = 0.10$(rad/s). Measurement noise of angular velocity v_2 was determined by specification of gyro sensor. We used $v_2 = 0.12$(rad/s).

$$\boxed{\begin{array}{ccc} \begin{pmatrix} q \\ \omega \end{pmatrix} & \longrightarrow & q' = q + T\,F'\,\omega + dq \\ \downarrow & & \downarrow \\ \begin{pmatrix} q^* \\ \omega^* \end{pmatrix} & \longrightarrow & q'^* = q^* + T\,F'\,\omega^* \end{array}}$$

We must determine v_1 with Polhemus delay $T = 80$ms taken into account. Let describe observed value by adding * and the value after T by adding $'$. For example, q'^* means the observed value of q after T. Suppose the human head rotate from q to q' within the interval dt. Suppose $q^* - q, \omega^* - \omega, dq$ be independent each other, and calculate the standard deviation of $q'^* - q'$. Let describe covariance matrix of $q'^* - q'$ by $\overline{(q'^* - q')(q'^* - q')^T}$.

$$\begin{aligned} &\overline{(q'^* - q')(q'^* - q')^T} \\ &= \overline{(q^* - q)(q^* - q)^T} + \overline{dq\,dq^T} \\ &\quad + dt^2\,\overline{F'(\omega^* - \omega)(\omega^* - \omega)^T F'^T} \end{aligned} \quad (34)$$

From

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$$

we approximate

$$F'F'^T \approx I \quad (35)$$

We also approximate $\overline{dq\,dq^T}$ by random walk.

$$\overline{(q^* - q)(q^* - q)^T} \equiv 0.0276^2 I_4 \quad (36)$$

$$\overline{dq\,dq^T} \equiv \frac{T}{dt} w_1^2 I_4 \quad (37)$$

$$\overline{F'(\omega^* - \omega)(\omega^* - \omega)^T F'^T} \approx \left(\frac{T}{dt} w_2^2 + v_2^2\right) I_4 \quad (38)$$

Finally we obtain $v_1 \sim 0.0508$.

C. *Result*

In order to make performance comparison easier to understand, we converted the estimated quaternion into ZYX Euler angle γ, β, α and compared various outputs with the standard signal(output of the link-type sensor). Fig.1 shows the standard signal, the raw output of Polhemus and the output of the proposed method. Average delay of Tracker was nearly 80ms. It is apparent that the proposed method compensated this delay well.

TABLE I

PERFORMANCE COMPARISON BY RMS ERROR. "POLHEMUS" DENOTES RAW DATA OF POLHEMUS TRACKER, "KALMAN" DENOTES KALMAN FILTERED RAW POLHEMUS DATA, AND "PROPOSED" DENOTES THE OUPTU OF PROPOSED METHOD.

	RMS (10^{-2} rad)		
	γ	β	α
Polhemus	7.12	9.22	9.00
Kalman	8.26	10.9	12.7
Proposed	7.10	4.52	4.32

1) *Evaluation by RMS error:* Table 1 shows the RMS(Root Mean Square) error of various outputs from the standard signal. RMS error of Kalman filtered raw Polhemus Tracker was greater than that of raw Tracker because of overshoots of its prediction. RMS error of the proposed method was reduced nearly half of raw Tracker output except roll angle. It was guessed that the arrangement of the metallic parts of the link-type sensor effected γ intensively.

2) *Evaluation by correlation technique:* Let x(t) be the original signal, and let y(t) be the measurement of $x(t)$ by a target sensor. Cross correlation function $\Phi_{xy}(\tau)$ [0] is defined as

$$\Phi_{xy}(\tau) \equiv \overline{x(t)y(t+\tau)} \quad (39)$$

$$\equiv \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)y(t+\tau)dt \quad (40)$$

The normalized cross correlation function $\rho_{xy}(\tau)$ [0] is defined by using $\Phi_{xy}(\tau)$ as

$$\rho_{xy}(\tau) \equiv \frac{\Phi_{xy}(\tau)}{\sqrt{\Phi_{xx}(0)}\sqrt{\Phi_{yy}(0)}} \quad (41)$$

This normalized cross correlation function has properties as below.

- If signal $y(t)$ follows $x(t)$ with lag of T(s) late,

$$x(t) \simeq y(t+T) \quad (42)$$

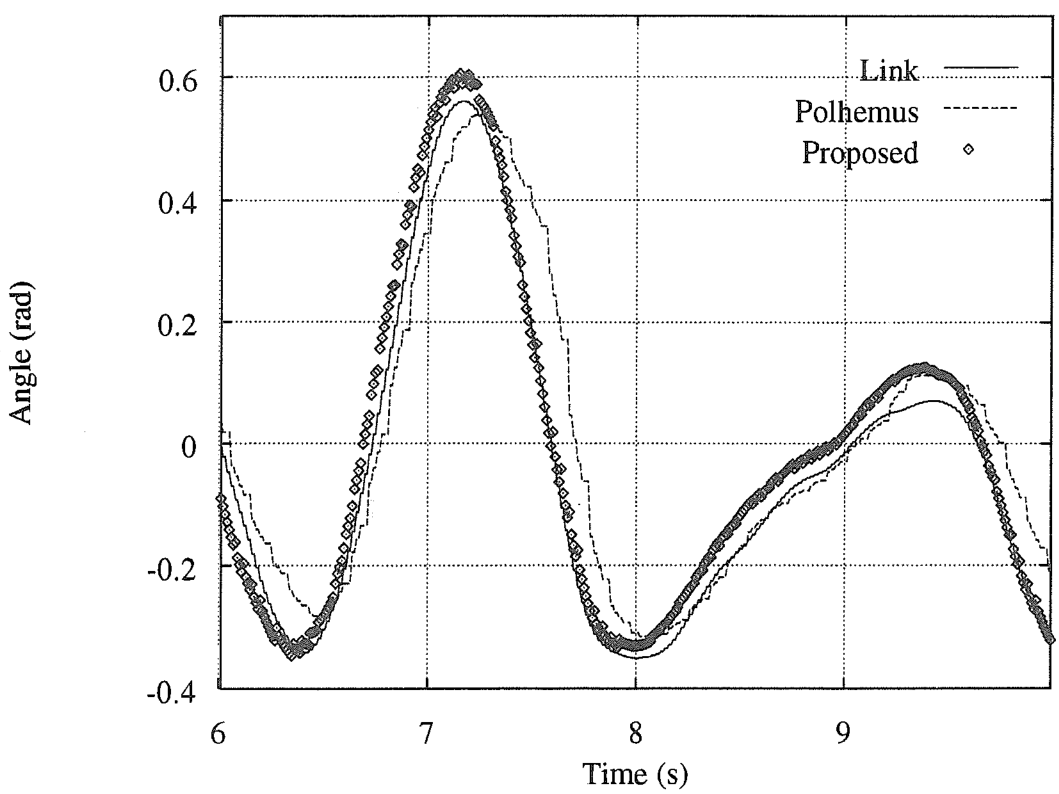

Fig. 2. Standard signal(outputs of link-type sensor), raw Polhemus Tracker data and the output of proposed method – roatation around axis Y of ZYX Euler angle

$\rho_{xy}(\tau)$ has a peak at $\tau \sim T$.

- The range of $\rho_{xy}(\tau)$ is .

$$-1 \le \rho_{xy}(\tau) \le 1 \tag{43}$$

- Suppose independent random noise be added on siganl $y(t)$. Let S be the signal power, N be the noise power. Then

$$\Phi_{xx}(0) = S \tag{44}$$
$$\Phi_{yy}(0) = S+N \tag{45}$$
$$\Phi_{xy}(T) = S \tag{46}$$

The peak of $\rho_{xy}(\tau)$ can be described

$$\rho_{xy}(T) = \frac{S}{\sqrt{\Phi_{xx}(0)}\sqrt{\Phi_{yy}(0)}} \tag{47}$$
$$= \sqrt{\frac{S}{S+N}} \tag{48}$$

$\rho_{xy}(T)$ is the function of SN ratio and can be the index of the fidelity of $y(t)$ to $x(t)$.

Therefore we can evaluate the delay and fidelity of the target sensor quantatively by checking the peak of its normalized cross correlation function [0].

Fig. 2 shows the $\rho(\tau)$ of Polhemus Tracker, Fastrak, and the proposed method. We used the output of mechanical link type sensor as the standard signal. The delay of Fastrak was half of Polhemus Tracker, but there remained still 30ms delay. There remained no delay for the proposed method, and its fidelity index was slightly improved.

IV. Real Time Implementation

The multi-sensor system consisted of a receiver of Polhemus Tracker and 3 compact gyro sensors (Piezo electric vibrating gyroscope GYROSTAR ENC-05S of Murata MFG Co.). This sensing system was attached to See-Through HMD. PC(Intel 80486 Overdrive 40MHz) read the measurement of Polhemus through RS232C (19.2Kbps), and that of gyro sensors via AD board.

In order to use the computational ability of PC efficiently as possible, we designed the interruption based program which can assign I/O waiting time to other tasks.

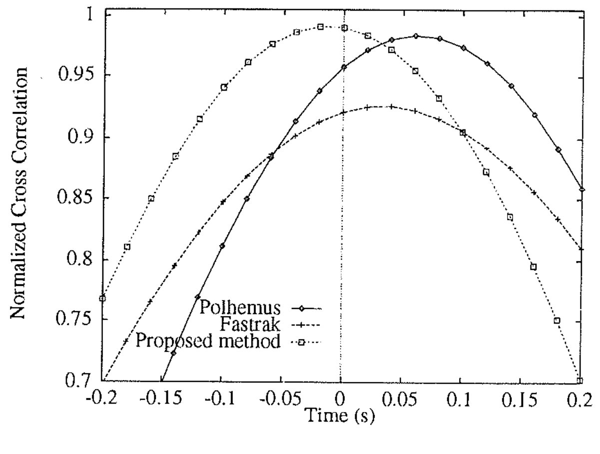

Fig. 3. $\rho(\tau)$ of Polhemus, Fastrak, the proposed method

The program consisted of 3 tasks below.

Task A Handling of Polhemus Tracker via RS232C

Task B Extrapolation based on the measurement of gyro sensors (Procedure 1)

Transmission of estimates via parallel printer port

Task C Optimal prediction based on the measurement of Polhemus and gyro sensors (Procedure 2)

We divided update calculation into two tasks. The reasons were: 1) the load of the calculation of Kalman filter was expected to be heavy, and 2) the unit sampling interval was critical for PC to finish that calculation within.

Task A was activated by RS232C interruption. Task B was activated every 10ms by interruption of DMA channel provided by AD board. The measurement of gyro was transferred by DMA channel. Task C, running in the background, requested present state vector to task B when the data from Polhemus was available, received it, calculated the algorithm of Kalman filter and sent the result to task B. Task B copied the result of task C if it was available, otherwise kept extrapolation based on the measurement of gyro.

3D rendering of virtual space was done on IBM PC by World Tool Kit(Sense8 Co.). IBM PC sent request to PC every image frame (10~12 frames/s). If requested, task B sent the latest estimate of human head motion to IBM PC via parallel port for printer.

We made a virtual copy of an actual room consisted of 49 polygons. 3D images of this room were updated 11 frames/s on average and communication between PC9801 and IBM PC via parallel port took only 1.4 ms. We checked synchronization between the virtual and the actual room by See-Through HMD and confirmed that not only the delay of Polhemus Tracker(~80ms) but also the delay of 3D rendering(~90ms) were well compensated and that we were able to obtain predicted orientation at 100Hz from Tracker measurement at 25Hz.

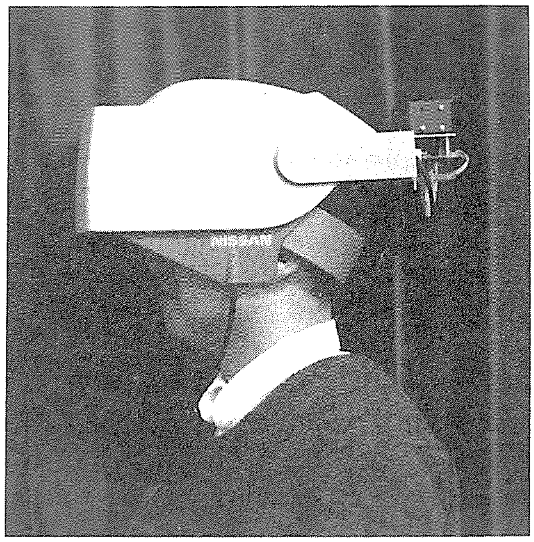

Fig. 4. See-Through HMD equipped with Polhemus and 3 compact gyros(in the box mounted at the rear of HMD)

V. Discussion

There are a lot of factors in VR systems which would cause time lag such as

- Dynamic response of the motion sensor and communication between the sensor and computer
- Communication between computers
- Computation for updating virtual space
- Computation of 3D rendering

Even if we could develop a sensor without any measurement delay, this would not solve all these problems. In this section we discuss the relation between our method and problems above. We also discuss the application of our method to tele-operation.

A. *Usage of multiple magnetic sensors*

In order to make user's commitment to virtual space more interactive, we should measure other parts of body such as hands and legs. The use of multiple magnetic motion sensors like Polhemus must be time sliced to avoid interference among themselves. The sampling rate for each motion sensor must be equally reduced. Our method is

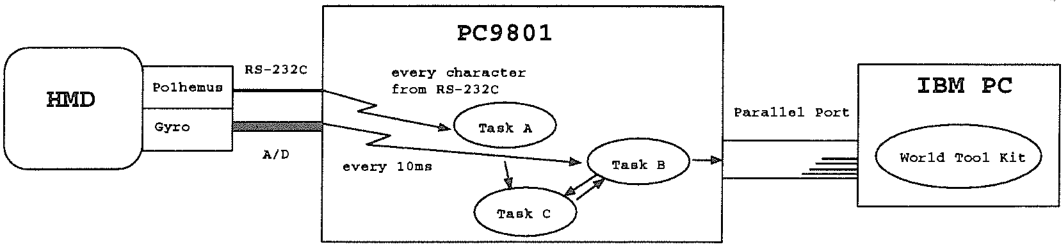

Fig. 5. Configuration and Software Architecture

useful to realize measurement of high sampling rate under this situation.

B. *Networked virtual environment*

In order to join people over distance through VR environment, the sites should be networked computers. It takes some time for changes in virtual space to propagate and to synchronize on the network. In each computer synchronization is realized by interruption process such as network arbitration, network control and low level system activity. It is expected that effects of such events will appear as time lags in non stationary manner. However fixed time lag compensation proposed in this paper could be the first order approximation of the final perfect solution to that problem.

C. *Delay by rendering*

The real-time generation of 3D images is one of the biggest problem in VR. It is obvious that 3D images, built up by complicated objects or processed by computationally heavy techniques like texture-mapping, give us better reality. The more complicated the virtual space is, the more it takes to render 3D images and the fewer the frame rate is. Frame rate by which we feel interactive is at least 10(frame/s) and maximum rendering delay can be estimated at 100ms. This must be taken into account. Therefore not only improvement of sensor but also fine prediction of human motion is essential for diminishment of total system delay.

D. *Application for teleoperation system*

In the situation where we connect an operator not to virtual space in computer but to a robot in other actual space(it is so called "advanced teleoperation" or "tele-existence"), we need to measure an operator's motion with much higher sampling rate ($\sim$ 1kHz) than that of Polhemus to control the robot smoothly. Our approach of multi-sensor integration is useful and essential to this problem.

VI. Conclusions

We proposed a multi-sensor integrated system which compensates time lag and raises sampling rate beyond the upper limit of Polhemus sensor. We confirmed the validity of the proposed method by off-line computation. We reported the real time implementation and discussed several reasons why our approach will be important for future VR systems.

References

[1] M.Friedmann, T.Starner and A.Pentland : *Syncronization in Virtual Realities*, PRESENCE, 1, 1, pp139-144, 1992.

[2] J.Liang, C.Shaw, and M.Green : *On Temporal-Spatial Realism in the Virtual Reality Environment* UIST '91, pp.19-25, 1990

[3] H.Goldstein : *Classical Mechanics 2nd ed.*, Addison-Weslay, 1980

[4] A.Gelb etc. : *Applied Optimal Estimation*, THE MIT Press, 1974

[5] S.Tachi, H.Arai and T.Maeda : *Tele-Existence Master Slave System For Remote Manipulation*, IROS '90, pp343-348, 1990

[6] K.G.Beauchamp : *Signal processing*, Geroge Allen and Unwin, 1973

[7] S.Emura and S.Tachi : *Sensor Fusion based Measurement of Human Head Motion*, Journal of the Robotics Society of Japan, vol.11, no.6, pp161-164, 1993 (in Japanese)

[8] S.Emura and S.Tachi : *Sensor Fusion based Measurement of Human Head Motion*, ROMAN94, 1994(*in press*)