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A TACTILE DISPLAY USING ULTRASONIC ELASTIC WAVES IN A METAL TAPERED MEMBRANE

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ABSTRACT

The purpose of the study is the development of a tactile display which can control a spatial pattern of friction on a metal surface by using ultrasonic elastic waves in a tapered membrane (a membrane whose width decreases gradually). It is reported that a surface of a vibrating object with ultrasonic frequency feels smooth by air lubrication called squeeze effect. In this paper, we show two properties of elastic waves in the tapered membrane. First, waves can be localized on the surface. Second, the localized vibrating area can be freely controlled(enlarged or reduced) by changing the frequency of the wave. Therefore, by generating ultrasonic elastic waves in the metal tapered membrane, we can construct a tactile display, where the localized smooth area can be moved on the metal surface. In an experimental display employing a brass-tapered membrane vibrated by a Langevin-type ultrasonic vibrator, validity of our method is confirmed.

INTRODUCTION

Haptic sensation is divided into two parts. One is proprioception, which is a sense of weight, force of resistance, or the approximate shape of an object. The other is tactile, cutaneous sensation, which is a sense of roughness, friction, or textures of an object's surface. The purpose of this study is to develop a tactile display that provides a tactile sensation, mainly about friction in active touch with a finger.

To generate varied tactile sensation, we cannot of course prepare all the real objects. Therefore it is necessary to control a shape of one object in real time. For controlling the state of the surface of the object, methods to use elastic waves have been proposed.

[1] propose a tactile display which controls friction of a metal plate using ultrasonic vibration. Standing waves of bending mode is generated on the metal plate by a Langevin-type ultrasonic vibrator. It is reported that the surface of the vibrating plate with ultrasonic frequency feels smooth by air lubrication called squeeze effect. Thus, ultrasonic elastic waves are used to control tactile sensation of friction.

On the other hand, [2] propose to use spatial amplitude modulated elastic waves. The wavelength and the group velocity of the envelope of the A.M. wave are freely controlled. Theoretically, an arbitrary surface shape is generated as the envelope of the spatial A.M. wave on the surface of an elastic plate. This changeable shape is used for tactile controlling.

However, the problem with these displays is that the standing wave or the A.M. wave cannot be localized in the plate. Therefore, during generating vibration, the surface is smooth everywhere on the plate.

In this paper, we propose a method for generating a distribution of waves for producing a more complicated spatial pattern of friction on a metal surface. We use a tapered membrane, that is the membrane whose width decreases gradually and continuously. It is shown that the wave in the tapered membrane can be localized at the wide end of the membrane. The area where the wave is trapped feels smooth by squeeze effect, and the area without the wave feels rough because of original metal friction. Thus, a smooth area and a frictional area can be created simultaneously on the metal surface. Furthermore, the boundary between these two areas is freely and continuously controlled by changing the frequency of the vibrator. With higher frequency, the smooth area is enlarged. Therefore, the spatial frictional state of the surface can be controlled by the frequency of the vibration.

An overview of this paper is as follows. First, we present an argument regarding elastic waves in a tapered membrane. This phenomenon is usually not the focus of detailed argument in elastic waves theory[3] [4], but is fundamental to our display.

Thus, an amplitude distribution of the elastic waves in the tapered membrane is analyzed at length. We show two advantages of the elastic waves in the tapered membrane for tactile controlling: 1)localization of waves and 2)controllability of the localized vibrating area. Using these properties of elastic waves in the tapered membrane, we propose a display which controls a spatial pattern of friction on the metal surface. We constructed an experimental device using a brass-tapered membrane and a Langevin-type ultrasonic vibrator. In the display, validity of our method is confirmed.

THEORY

Derivation of a equation of elastic waves in a tapered membrane

A membrane with a variable width according to $y = W(x)$ is considered.(Figure 1) We call a membrane whose width decreases monotonically and gradually 'tapered membrane'. The membrane is tensed, and clamped at the edges; $y = W(x)$.

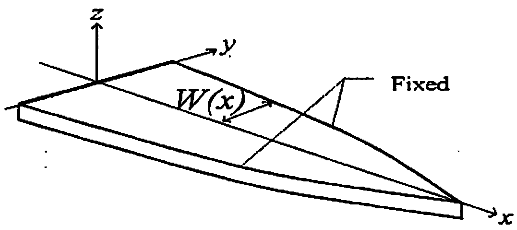

Figure 1: Tapered membrane

We set the x axis along the direction which the wave progresses, the y axis for the direction of the width of the membrane, and the z axis perpendicular to the other axes. T is tension of the membrane, and ρ is density of the membrane.

A displacement for the z direction denoted v satisfies the wave equation and fixed boundary conditions at the edges:

$$\frac{1}{V^2}\frac{\partial^2 v}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)v, \quad (1)$$

$$\text{where } V = \sqrt{\frac{T}{\rho}}. \quad (2)$$

$$v = 0 \quad \text{at} \quad y = \pm W(x). \quad (3)$$

Now, curvilinear coordinates (X, Y) defined as follows are introduced:

$$X = x, \quad (4)$$

$$Y = \frac{y}{W(x)}. \quad (5)$$

Then the next lemma is proved (Apendix).

Lemma If $W\prime(x) << 1$, that is, if a changing rate of the width of the membrane is small enough, (X, Y) is a rectangular coordinate.–

In (X, Y) coordinate, the wave equation(1) and the boundary conditions(3) become

$$\frac{1}{V^2}\frac{\partial^2 v}{\partial t^2} = \left(\frac{\partial^2}{\partial X^2}+\frac{1}{W^2(X)}\frac{\partial^2}{\partial Y^2}\right)v, \quad (6)$$

$$v(X, Y) = 0 \quad \text{at} \quad Y = \pm 1. \quad (7)$$

The boundary conditions (7) are independent of X. In this way, using (X, Y) coordinates, a problem in the tapered membrane is translated into a problem in the straight membrane.

Considering fixed boundary conditions in (7), a separable solution,

$$v(X, Y) = f(X)\cos\frac{\pi Y}{2}e^{-j\omega t} \quad (8)$$

can be assumed. $f(X)$ represents an amplitude distribution along the x axis, and this is what we want here. Substituting this solution into the equation(6), we obtain

$$\frac{d^2 f(X)}{dX^2}+\left(\left(\frac{\omega}{V}\right)^2-\left(\frac{\pi}{2W(X)}\right)^2\right)f(X) = 0. \quad (9)$$

By restoring the variables (X, Y) into (x, y), after all we obtain the amplitude in the tapered membrane as

$$v(x, y) = f(x)\cos\frac{\pi y}{2W(x)}e^{-j\omega t}, \quad (10)$$

where

$$\frac{d^2 f(x)}{dx^2}+\left(\left(\frac{\omega}{V}\right)^2-\left(\frac{\pi}{2W(x)}\right)^2\right)f(x) = 0. \quad (11)$$

We call the equation(11) the tapered membrane equation.

Relation between the tapered membrane equation and the Schrödinger equation

Now, we compare the tapered membrane equation (11) with the Schrödinger equation for a stationary state in one dimension in quantum mechanics:

$$\frac{d^2\phi(x)}{dx^2}+\frac{2m}{\hbar^2}(E - V(x))\phi(x) = 0. \quad (12)$$

A probability amplitude ϕ in the Schrödinger equation corresponds to the real amplitude $f(x)$ of the membrane in the tapered membrane equation. The energy of a system E in quantum mechanics corresponds to the energy of the elastic wave which is proportional to the square of the frequency of the wave. Then we can introduce an important concept of the potential energy of the elastic membrane, which corresponds to the potential $V(x)$ in the Schrödinger equation. The potential for elastic waves is inversely proportional to the square of the width of the membrane.

This comparison might seem to be somewhat abrupt, but by this analogy, it becomes easier to understand a behavioral pattern of elastic waves in the tapered plate. The problem to input elastic waves from the origin toward the right side with frequency ω is equivalent to a scattering problem in one dimension(Figure 2) with energy of $\left(\frac{\omega}{V}\right)^2$ toward the potential of $\left(\frac{\pi}{2W(x)}\right)^2$.

(a)Schrödinger equation		(b)tapered membrane equation
Input energy: E	$\leftrightarrow$	$\left(\frac{\omega}{V}\right)^2$
Potential energy: $V(x)$	$\leftrightarrow$	$\left(\frac{\pi}{2W(x)}\right)^2$

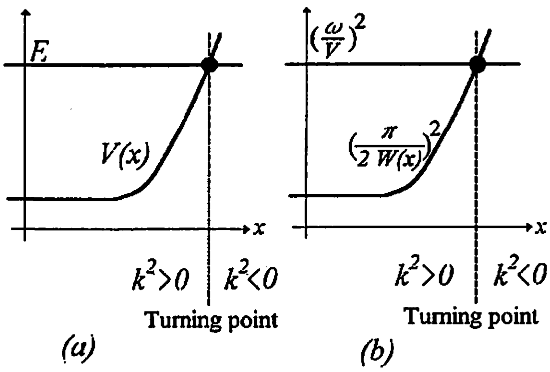

Figure 2: Analogy between (a)the Schrödinger equation and (b)the tapered membrane equation. The tapered width $W(x)$ produces a monotone increasing potential

Localization of the wave

The property of the wave changes drastically at the point where the energy of the wave is equal to the potential. This point is called the turning point(Figure 2).

From the origin to the turning point, the local wavenumber defined as

$$k(x)^2 = \left(\frac{\omega}{V}\right)^2 - \left(\frac{\pi}{2W(x)}\right)^2 \tag{13}$$

is real. Thus, in this area, the wave is sinusoidal.

However, outside the turning point, the energy of the wave is smaller than the potential. Therefore the wavenumber becomes imaginary. This means the amplitude of the wave attenuates exponentially.

In this way, the wave is localized at the wide end of the membrane, that is, from the origin where the wave is input to the turning point.

Analytical expression of the amplitude distribution of the elastic waves in the tapered membrane

Now, the tapered membrane with the width

$$W(x) = W_0 - dx, \tag{14}$$

is considered. the potential and the local wavenumber become

$$V(x) = \left(\frac{\pi}{2(W_0 - dx)}\right)^2 \tag{15}$$

$$k^2(x) = \left(\frac{\omega}{V}\right)^2 - \left(\frac{\pi}{2(W_0 - dx)}\right)^2. \tag{16}$$

Then the position of the turning point defined as $k^2(x_0) = 0$ is

$$x_0 = \frac{1}{d}\left(W_0 - \frac{\pi V}{2\omega}\right). \tag{17}$$

From the assumption that $W\prime(x) << 1$, we expand the wavenumber (16) around the turning point (17) as a linear function of x as follows:

$$\begin{aligned} k^2(x) &= k^2(x_0) + \frac{dk^2(x_0)}{dx}(x - x_0) \\ &= -k_0^2(x - x_0), \end{aligned} \tag{18}$$

$$\text{where } k_0^2 = \frac{4d\omega}{\pi V}, \tag{19}$$

Then the tapered membrane equation(11) turns into

$$\frac{d^2 f(x)}{dx^2} - k_0^2(x - x_0)f(x) = 0. \tag{20}$$

This equation has an analytical solution as follows:[3]

$$f(x) = A\frac{\pi}{\sqrt{3}}\sqrt{x_0 - x}\left(J_{\frac{1}{3}}\left(\frac{2}{3}k_0(x_0 - x)^{\frac{3}{2}}\right) + J_{-\frac{1}{3}}\left(\frac{2}{3}k_0(x_0 - x)^{\frac{3}{2}}\right)\right) \quad (x < x_0) \tag{21}$$

$$f(x) = A\sqrt{x - x_0}K_{\frac{1}{3}}\left(\frac{2}{3}k_0(x - x_0)^{\frac{3}{2}}\right) \quad (x > x_0), \tag{22}$$

where A is a constant determined by the input.

For example, the amplitude distribution in the brass-tapered membrane is shown in Figure 3, where $\rho = 8.11$[g/cm^3], $T = 5.12 \times 10^{10}$ [dyne/cm^2], $W_0 = 3$[cm], $d = 0.1$, $f = 19.6$[kHz].

From the origin to the turning point, a sinusoidal amplitude increases monotonically. Then at the turning point, the wave attenuates exponentially. The wave is localized at $0 < x < x_0$.

Control of the localized vibrating area

In this section, we show that the vibrating area localized at the wide end of the tapered membrane can be enlarged or reduced by the frequency of the wave. The change of the position of the turning point in a brass membrane is shown in Figure 5.

We consider this problem with the potential again(Figure 4). By changing the frequency of the wave, the energy of the wave changes according to $\left(\frac{\omega}{V}\right)^2$. The turning point is a crossing point of the curve of the potential and the line of the energy. Thus, the turning point moves from x_L to x_H when the frequency changes from ω_L to ω_H. In this way, the vibrating area is enlarged by increasing the frequency.

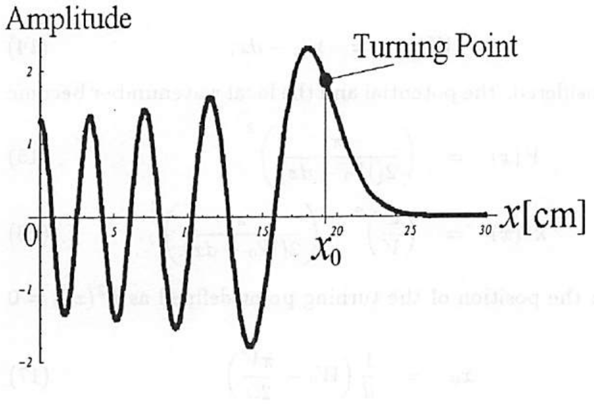

Figure 3: amplitude distribution in the tapered membrane. The amplitude is normalized by the amplitude at the origin

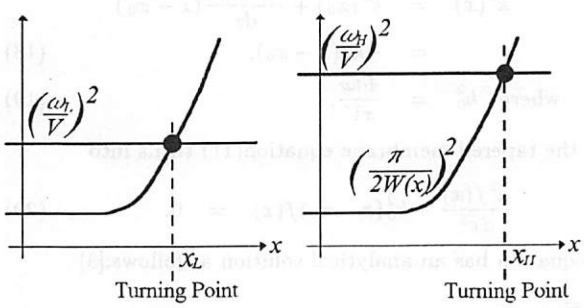

Figure 4: The turning point moves from x_L to x_H when the frequency changes from ω_L to ω_H.

The properties of the wave in the tapered membrane

From the above discussion, the wave in the tapered membrane has the following two properties:

Property 1) The wave is localized from the entrance of the wave to the turning point.

Property 2) The position of the turning point which is the boundary of the localized vibrating area can be controlled by the frequency of the wave.

We propose a tactile display using these properties in the next section.

Method for tactile controlling

It is reported that a vibrating elastic plate with ultrasonic frequency feels smooth by air lubrication called squeeze effect [1]. This sensation is obtained by only touching the surface of a Langevin-type ultrasonic vibrator.

Now, we input an ultrasonic wave into a tapered membrane. From the property 1) of the wave, the sinusoidal vibration is localized at the wide end of the membrane. The vibrating area feels smooth, but the non-vibrating area feels rough by original friction of the metal. Therefore, smooth and frictional areas are created simultaneously in the surface. Furthermore, from

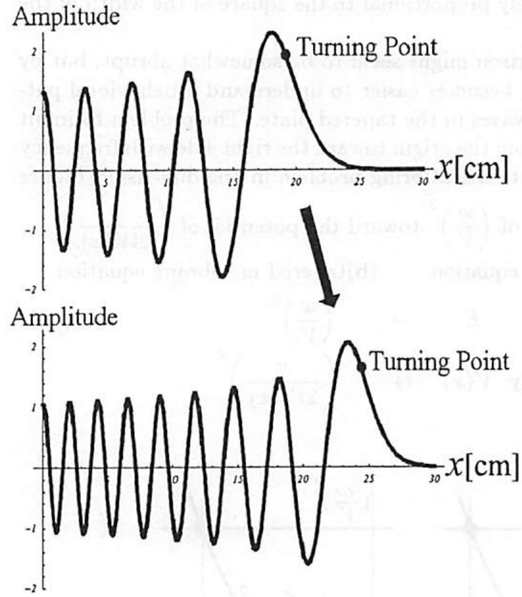

Figure 5: Control of the turning point by the frequency (a)f=20[kHz], (b)f=40[kHz]

the property 2), the smooth area can be enlarged or reduced continuously by changing the frequency of the wave.

In this way, a spatial distribution of friction on the metal surface can be controlled in real time. This is our method for tactile controlling by ultrasonic elastic waves in the metal tapered membrane.

EXPERIMENT

Basic experiment

To examine the basic phenomena of elastic waves in the tapered membrane, a basic experiment is conducted first. The experimental system is shown in Figure 6.

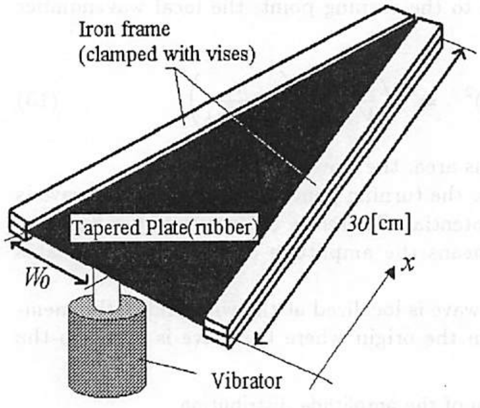

Figure 6: Basic experiment using rubber tapered membrane

To obtain a large amplitude of the wave, we used a rubber-membrane and a voice coil with auditory frequency. The thickness of the membrane is 0.02[mm], and the width is 100[mm]. The rubber-membrane is clamped by the tapered iron plate, thus creating fixed boundary conditions.

For measurement of vibrations, a laser displacement meter (Keyence LC-2440) is used.

A spatio-temporal distribution of the amplitude of the wave is shown in Figure 7(a). The graph is drawn when the frequency is 30[Hz]. The x axis is the position in the membrane, t axis is the temporal axis for one cycle (1/30[sec]), the vertical axis is the amplitude of the wave. There is a boundary at about $x = 15$. In the region, $0 < x < 15$, the temporal wave is sinusoidal (Figure 7(b)). On the contrary, in the region, $x > 15$, the membrane does not vibrate for all the time (Figure 7(c)). This shows that the turning point surely exists at about $x = 15$.

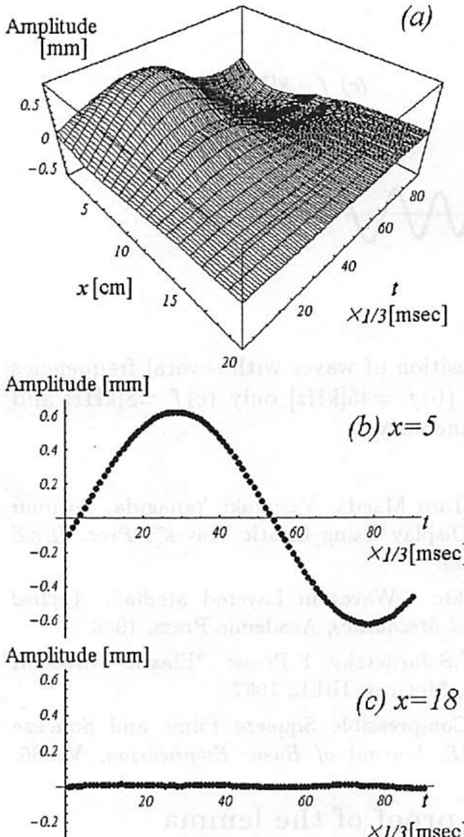

Figure 7: Experimental result 1: (a) Distribution of the spatio-temporal displacement in the tapered membrane. (b) Displacement at $x = 5$ (c) Displacement at $x = 18$ In the region, $x > 15$, the wave attenuates for all the time.

In Figure 8, it is shown that the position of the turning point is changed by the frequency. Figure (a) shows the distribution of the amplitude in 30[Hz], and Figure (b) shows that in 40[Hz]. As the theory shows, the wave with higher frequency reaches further from origin. The position of the turning point is controlled by the frequency.

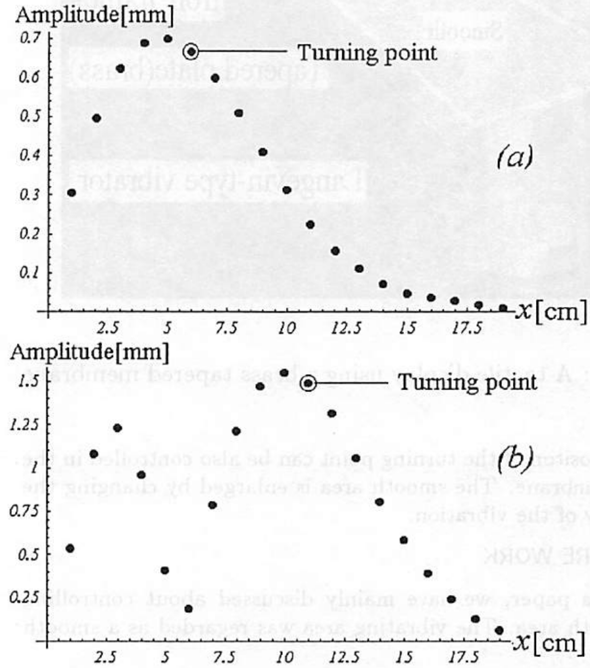

Figure 8: Experimental result 2: Maximum value of the amplitude at each point in the rubber-tapered membrane (a) f =30[Hz], (b) f =40[Hz]

A tactile display of brass-tapered membrane

Based on the basic experiment, we construct a display of brass-tapered membrane vibrated with ultrasonic frequency (Figure 9).

As a vibrator, a bolt-clamped Langevin-type ultrasonic vibrator (NTK D4520PC) is used. The resonant frequency is 19.6[kHz]. The thickness of the membrane is 0.05[mm]. The tapered iron frames are clamped by vises.

Without inputting waves, we feel friction of the brass everywhere on the surface. We sometimes feel a stick-slip force.

With generating the ultrasonic vibration, the friction around the wide end of the membrane becomes lower obviously, though the surface around the narrow end of the membrane remains frictional. This shows that the turning point where the wave attenuates also exists in the brass-membrane. From the origin to the turning point, the surface feels smooth by squeeze effect. From the turning point to the narrow end of the membrane, the surface feels rough.

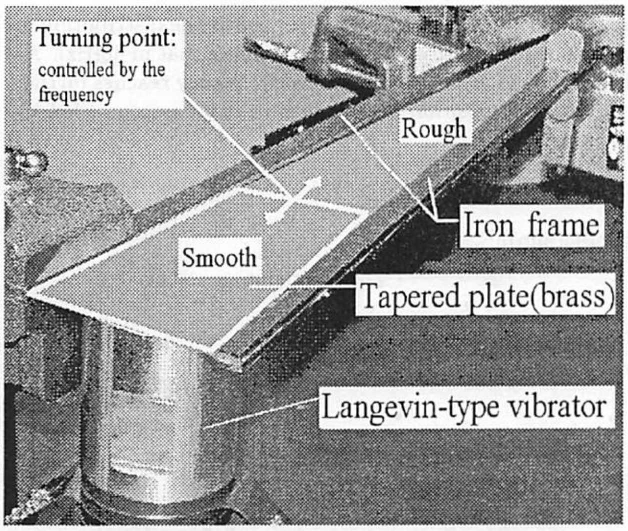

Figure 9: A tactile display using a brass tapered membrane

The positon of the turning point can be also controlled in the brass-membrane. The smooth area is enlarged by changing the frequency of the vibration.

FUTURE WORK

In this paper, we have mainly discussed about controlling the smooth area. The vibrating area was regarded as a smooth surface.

Squeeze force that provides the smoothness depends on an amplitude of the vibration[5]. Thus, if a distribution of squeeze force can be created, a more complicated distribution of friction can be generated.

Here, waves in the tapered membrane has the following remarkable property for tactile controlling besides two properties mentioned in this paper.

Property 3) Several waves with several frequencies are superposed in the linear elastic membrane. -

A position of a peak of an amplitude is determined by the frequency. Thus, when the membrane is vibrated with several frequencies simultaneously, several peaks are formed on the surface of the membrane. For an example, we show an distribution of the amplitude with 8[kHz](Figure 10(a)), 45[kHz] (Figure (b)), 8[kHz] and 45[kHz] simultaneously(Figure (c)). There are several peaks in the membrane. This means we can create the distribution of squeeze force. We will examine whether or not the distribution of squeeze force in finger pad affects tactile sensation in the near future.

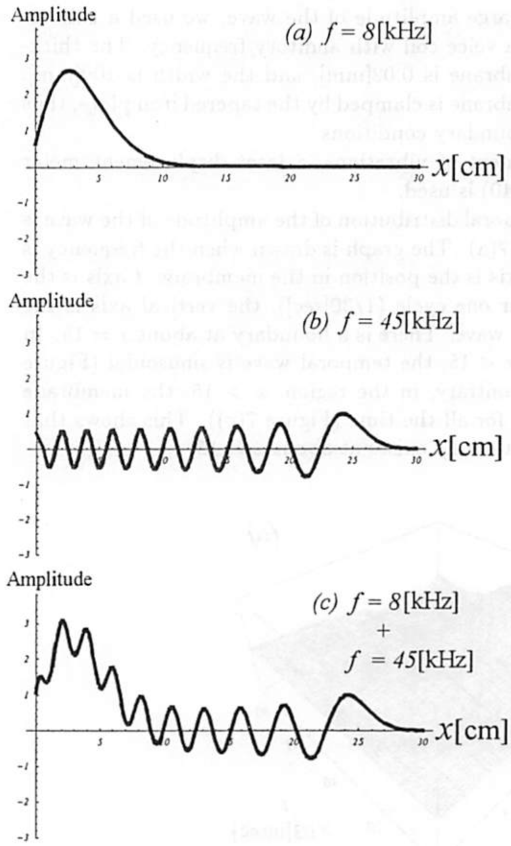

Figure 10: Superposition of waves with several frequencies (a)f =8[kHz] only (b)f =45[kHz] only (c)f =8[kHz] and f =45[kHz] simultaneously

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Appendix : A proof of the lemma

$$\begin{aligned} \nabla X \cdot \nabla Y &= \left(\frac{\partial X}{\partial x}, \frac{\partial X}{\partial z}\right) \cdot \left(\frac{\partial Y}{\partial x}, \frac{\partial Y}{\partial z}\right) \\ &= (1, 0) \cdot \left(-z\frac{W'}{W^2}, \frac{1}{W}\right) \\ &= -z\frac{W'}{W^2} \\ &= 0 \quad \text{Q.E.D} \end{aligned}$$

Thus, (X, Y) is a rectangular coordinate.