Measurement of Auditory Alley in Virtual Environment and Its Mathematical Model

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ABSTRACT

In space perception, the straight lines running phenomenally parallel to the median plane and straight lines running phenomenally equi-distant from the median plane are called parallel alley and equi-distance alley, respectively. It is known that these lines are concave bending towards the median plane in binocular visual space. Using virtual environmental display system, it became possible to construct the experimental system to measure the parallel alley and the eqi-distance alley in binaural auditory space. The experiments were conducted and the results were obtained as follows:

- (1) Parallel alley and eqi-distance alley in auditory space are not always straight in physical sense, and the forms of them depend on the distance from the median plane.
- (2) Parallel alley lies inside eqi-distance alley.

Employing sound intensity and binaural time differences as parameters, mathematical models were constructed to explain the auditory alleys. By simulation, it was confirmed that the models successfully explain the experimental results obtained in virtual environmental system.

1. INTRODUCTION

For binocular visual space, it is well-known that horizontal lines which appear straight on a subjective frontoparallel plane are not always straight in the physical sense, and that the form of the lines depends on the distance from the observer. This phenomenon is known as Helmholtz's horopter[1]. On the other hand, parallel alley and equidistance alley represent the concept of subjectively parallel to the median plane. In Fig.1, suppose that x axis is the intersection of the horizontal plane of the eye level and the median plane of the observer, and y axis is the intersection



of the same horizontal plane and the frontoparallel plane including two eyes of the observer. A pair of lines p shows parallel alley and the other pair of lines d shows equidistance alley. Parallel alley is a pair of perceived straight lines obtained when two rows of luminous points on the horizontal plane of the eye level are arranged symmetrically to appear straight and parallel to the median plane. Physically, the two lines should be straight and parallel to each other, but they are actually not and their form depends on the distance from the median plane. Equidistance alley is a pair of perceived straight lines obtained when two rows of luminous points on horizontal plane of the eve level are arranged symmetrically to appear straight and laterally equi-distant from the median plane. Physically, the two lines should be straight and equi-distant, but they are actually not and their form depends on the distance from the median plane[2]. Moreover, the two kinds of alleys should be physically the same, but they are perceptually not and usually the latter lies outside the former in physical space as shown in Fig.1[3]. A similar phenomenon is also known for tactile space[4][5].

A basic experiment for sound localization with distance was conducted by the authors, and a similar phenomenon to Helmholz's horopter was found in binocular auditory space. That is, it was confirmed that the forms of the lines subjectively parallel to frontoparallel plane are not always straight in the physical sense and they depend on the distance between the sound source and the subject[6]. In this experiment, sound intensity and binaural time differences are considered to be very important cues for the subjects to judge the position of the sound sources. Then, these two cues were employed as parameters to formulate a mathematical model for space perception to explain the auditory horopter, and the formulated model was confirmed by simulation to be able to explain the distance dependency of the forms of auditory horopter[7].

From the similarity of sensory modalities and the phenomenon of auditory horopter, phenomenon of parallel alley and equi-distance alley in auditory similar to visual space and tactile space is expected. There has been no study, however. In order to measure the form of the perceptual parallel lines to the median plane in binaural auditory space, several sound sources should be located symmetrically with the median plane, and the sound from each sound source should be outputted in order. Assuming such an experimental system setting up actually, the space for the system must be large, and the physical existence of nearer sound sources must occlude the sound from farther sound sources to reach the ears of the subjects. These difficulties may prevent to carry the experiments for auditory alley. Then, the authors has tried to realize the psychophysical experimental system to measure the forms of parallel alley and equi-distance alley in auditory space by using virtual environmental display system which was composed for study of sound distance perception[8].

2. THE MEASUREMENT OF PARALLEL AND EQUI-DISTANCE ALLEYS IN AUDITORY SPACE 2. 1 The System Setup

The diagram of the virtual environmental display system used in this study is shown in Fig.2. This system is the same as the one used in the study to clarify the role of some cues for the sound distance localization by using virtual environment[8].

In order to construct the virtual environment, two computers(IRIS Indigo) were used. By the computers, computer graphics images for both eyes were generated



Fig.2 Virtual environmental display system

and served as the visual signals for the subject through a head-mounted display attached to the subject. As for headmounted display, an see-through head-mounted display(STHMD), which superimposes the virtual environment on the real environment by beam splitter, was employed. A calibration method was developed and applied to the visual parameters of the STHMD to eliminate errors caused by mechanical misalignments in the HMD and by individual differences between actual and designed location of pupils of eyes, which ensured the coincidence of the apparent distance in virtual environment with that in real environment[9]. By using one of the computers, any auditory signals were synthesized, outputted through DSP, and displayed through a headphone (Senheiser HD25SP) to both ears of the subject.

2.2 Experimental Method

We designed the experiments to measure parallel alley and equi-distance alley in auditory space using the method of constant stimuli[10]. The diagrams of the experimental systems are shown in Fig.3.

A pair of two sound sources in the symmetrical position to the median plane of the subject is called "pair of sound sources" hereinafter. Four virtual sound sources are assumed, which consist of a standard pair of sound sources and a comparing pair of sound sources, and the latter pair is located nearer to the subject than the former pair with various inner distances. As visual signals, images of two sound sources which are in the symmetrical position of the median plane on the horizontal plane of ear level of the subject are displayed as the images of the standard pair of the sound sources. The auditory signal given to the both



Fig.3 Experimental system of parallel alley and equi-distance alley

ears of the subject is pseudo random noise, and its intensity and binaural time differences are controlled as follows: Consider the sound stimuli which reaches both ears of the subject from a sound source located at the left side of the median plane. The following notation will be used:

- P_l : The intensity of the sound stimuli which reaches the left ear directly from the objective sound source
- *d* : The distance between the objective sound source and the subject
- P_0 : The standard sound intensity
- d_0 : The distance between the subject and the sound source from which the sound stimuli reached directly the left ear has the standard sound intensity.

As P_i is anti-proportional to the distance from the sound source[11], the sound stimuli reaches the left ear of the subject is

$$P_l = \frac{d_0}{d} p_0 \ . \tag{1}$$

On the other hand, the sound stimuli reaches the right ear via the circumference of the head of the subject as shown in Fig. 4. Therefore, the difference between the distance from the sound source to the right ear and the distance to the left ear (Δd) is approximated as follows, where the radius of the head of the subject is r [12].

$$\Delta d = r\theta + r\sin\theta \,. \tag{2}$$

The angle θ is executed by using the distance from the sound source (y) and the distance between the sound source and the median plane (x).

$$\theta = \arcsin(x / y). \tag{3}$$

Therefore, the intensity of the sound stimuli reaches the



Fig.4 Approximation of the difference between sound source and both ears

right ear (P_r) is executed as follows:

$$P_r = \frac{d_0}{d_0 + \Delta d} p_l. \tag{4}$$

Also, the time difference between the sound stimuli reached the right ear and the left ear (Δt) is executed as follows, where r is 8.75 cm and the sound velocity (v) is 340m/sec:

$$\Delta t = \Delta d / v = 257(\theta + \sin \theta) \ [\mu \, \text{sec}]. \tag{5}$$

Assuming the sound stimuli from the left sound source, the auditory signal given to the right ear of the subject is controlled to have a ratio and a binaural time difference with the signal given to the left ear using the above equations(4),(5). Assuming the sound stimuli from the right sound source, the auditory signals are given to both ears oppositely.

The head of the subject is fixed and visual parameters of STHMD are calibrated individually before the experiments. Each trial is composed as follows:

(1) By assuming that the standard pair of the virtual sound sources are located at a certain distance from both ears of the subject, parallel to the physical median plane, and symmetric with regard to the plane, the sound stimuli supposed from the right source of the standard pair of the sound sources and the stimuli supposed from the left one are given to the subject in order repeatedly with 500msec-duration and 500msec-interval. A subject is instructed to move a pair of visual markers of the virtual sound sources to the position where they are perceived by using a mouse cursor in order to fix the



Fig.5 Order of display and relations between two pairs of virtual sound sources in experiments of parallel alley



Fig.6 Order of display and relations between two pairs of virtual sound sources in experiments of equi-distance alley

subjective position of the standard pair of the sound sources. After that, the position of the pair of visual markers are consistent so that the subject can confirm the position visually.

(2) Next, the sound stimuli assuming from the standard pair and the comparing pair of the sound sources are given with 500msec-duration and 2500msec-interval repeatedly in order, and the subject is indicated to reply the result of his/her judgement by using two mouse buttons. In case of experiments to measure the form of parallel alley, the order of the virtual sound sources of sound stimuli is assumed as right standard sound source, right comparing sound source, left standard sound source, and left comparing sound source as shown in Fig.5(a). The subject judges the relation between the line connecting the former two sound sources is whether type(b) or type(c) shown in Fig.5. On the other hand, in case of experiments to measure the form of the equi-



Fig.7 Results of analysis

distance alley, the order of the virtual sound sources of sound stimuli is assumed as right standard sound source, left standard sound source, right comparing sound source, and left comparing sound source as shown in Fig.6(a). The subject judges whether the inner distance of the latter two sound sources is more narrow or wider than that of the former two sound sources as shown in Fig.6(b) and (c).

The assuming distance and inner distance of the standard pair of the sound sources are $4\sqrt{2}$ m, and 0.4 m respectively. The assuming distance of the comparing pair of the sound sources is set as $2\sqrt{2}$ or 4 m, and their assuming inner distance is set as 0.0, 0.2, 0.4, 0.6, or 0.8 m.

2.3 The Experimental Results and Their Analysis

In one trial of the experiments to measure the form of the parallel alley or the equi-distance alley, the sound stimuli were presented to a subject with randomly selected inner distance of the comparing pair of the sound sources, where every inner distance was assumed four times each. As the results of each trial, psychometric functions were obtained, and probit analysis[13] was applied to estimate the equivalence of the inner distance of the comparing pair of the sound sources, which was perceived to be parallel to or equi-distant with the standard pair of the sound sources. One example of the results of the analysis is shown in Fig.7, where white dots show the experimental results of the measurement of the parallel alley, and black dots show the experimental results of the measurement of the equidistance alley.

From the results of the analysis, followings are obtained as the general tendencies:

- (1) Both parallel alley and equi-distance alley in auditory space have different form with that of physically parallel line to the median plane.
- (2) The auditory parallel alley lies inside the auditory equidistance alley in physical space.

These tendencies are similar to the relations between parallel alley and equi-distance alley in visual space or tactile space obtained in real environment.

3. THE MATHEMATICAL MODEL FOR SPACE PERCEPTION TO EXPLAIN AUDITORY ALLEYS

3.1 The Formulation of the Mathematical Model for Space Perception

We, the authors, have constructed a neural network model called ISLES model (Independent Scalar Learning Element Summation Model) using biological information and constraints, and the model have succeeded to explain the distance dependency of the forms of horopter and alleys in visual space and in tactile space uniformly[14]. Moreover, it was confirmed that the distance dependency of the auditory horopter can be successfully explained by the ISLES model constructed by using sound intensity and binaural time difference as parameters[7]. Therefore, it is attempted to construct the ISLES model which explains auditory alleys by using sound intensity and binaural time difference as parameters.

Based on [15], sound intensity k is defined as the product of the sound intensity reached to the right ear I_A and the sound intensity reached to the left ear I_B ,

$$k = I_A \cdot I_B . (6)$$

Then, the origin of a coordinate system in the physical space is set to be the midpoint of both ears, x axis is set to be the line including both ears, and y axis is set to be the horizontal line perpendicular to the x axis. The sound intensity k of a certain position (x, y) in the coordinate system is represented as follows:

$$(x^{2} + y^{2} + a^{2})^{2} = 4a^{2}x^{2} + \frac{I_{s}^{2}}{k}, \qquad (7)$$

where I_s is the intensity of the sound source, and a is

the distance from the origin to both ears[15]. Also, as for binaural time differences Δt , the following equation is obtained from the equation of phase difference described in [15]:

$$\Delta t = \frac{\sqrt{(x+a)^2 + y^2} - \sqrt{(x-a)^2 + y^2}}{v},$$
(8)

where v is the sound velocity.

As the model for auditory parallel alley, the following equation is assumed on the analogy of the model for auditory horopter:

$$\hat{H}_{p}(k,\Delta t) = \Delta t + \hat{H}_{t}(k), \qquad (9)$$

where binaural time differences (Δt) is compensated by the function of sound intensity(k).

Assuming the learning area for auditory space perception is from x_{\min} to x_{\max} to the direction of distance from the median plane (x), and the distribution of learning points is uniform concerning with x, the second term of the above equation(9) is represented as follows:

$$\hat{H}_t(k) = -\frac{\int_{x_{\min}}^{x_{\max}} \Delta t(x,k) dx}{x_{\max} - x_{\min}} .$$
(10)

On the other hand, the ISLES model for auditory equidistance alley is assumed as follows:

$$\hat{H}_{d}(k,\Delta t) = \hat{H}_{u}(k) + \hat{H}_{c}(\Delta t).$$
(11)

Assuming the learning area is from y_{min} to y_{max} to the direction of distance from the frontoparallel plane including both ears (y) and the distribution of learning points is uniform concerning with y, the following equations are obtained similar to the equation (10):

$$\hat{H}_{u}(k) = \frac{\int_{y_{\min}}^{y_{\max}} x(y, k) dy}{y_{\max} - y_{\min}},$$
(12)

$$\hat{H}_{c}(\Delta t) = \frac{\int_{y_{\min}}^{y_{\max}} x(y, \Delta t) dy}{y_{\max} - y_{\min}}.$$
(13)

3.2 The Examination of the Experimental Results

By executing equations (10), (12), and (13) for various learning areas, the forms of the equations (9) (parallel array) and (11) (equi-distance alley) are examined. As the results of these simulations, it is confirmed that these forms



Fig.8 Examples of the results of simulation

have similar tendencies with the experimental results of auditory alleys by using virtual environment.

Examples are shown in Fig.8 ($x_{\min} = 0$ [mm], $x_{\max} = 300$ [mm] for parallel alley, and $y_{\min} = 1000$ [mm], $y_{\max} = 18000$ [mm] for equi-distance alley).

4. THE CONCLUSION

By using virtual environmental display system, experimental system to measure parallel alley and equidistance alley in auditory space is constructed.

As the results of the experiments using the system, the followings are obtained:

(1) The forms of parallel alley and eqi-distance alley in auditory space are not always straight in physical sense, and depend on the distance from the frontoparallel plane.

(2) Parallel alley lies inside the eqi-distance alley.

These results are in good agreements with the phenomena in binocular visual space and tactile space.

Employing sound intensity and binaural time differences as parameters, mathematical models were constructed to explain the auditory alleys. The models were based on the mathematical model using scalar learning rule proposed by T. Maeda et al.[14]. By simulation, it was confirmed that the models successfully explain the experimental results obtained by using virtual environmental system.

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